

# A HISTORICAL WELFARE ANALYSIS OF SOCIAL SECURITY: WHOM DID THE PROGRAM BENEFIT?<sup>1</sup>

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Abstract A well-established result in the literature is that Social Security reduces steady state welfare in a standard life cycle model. However, less is known about the historical quantitative effects of the program on agents who were alive when the program was adopted. In a computational life cycle model that simulates the Great Depression and the enactment of Social Security, this paper quantifies the welfare effects of the program's enactment on the cohorts of agents who experienced it. In contrast to the standard steady state results, we find that the adoption of the original Social Security generally improved these cohorts' welfare, in part because these cohorts received far more benefits relative to their contributions than they would have received if they lived their entire life in the steady state with Social Security. Moreover, the negative general equilibrium welfare effect of Social Security associated with capital crowd-out was reduced during the transition, because it took many periods for agents to adjust their savings levels in response to the program's adoption. The positive welfare effect experienced by these transitional agents offers one explanation for why the program that may reduce welfare in the steady state was originally adopted.

KEYWORDS: Social Security, Recessions, Great Depression, Overlapping Generations.

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*“We can never insure one hundred percent of the population against one hundred percent of the hazards and vicissitudes of life, but we have tried to frame a law which will give some measure of protection to the average citizen and to his family against the loss of a job and against poverty-ridden old age.”*

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F.D. Roosevelt during the signing of The Social Security Act of 1935

## 1. INTRODUCTION

Social Security was implemented in the midst of the Great Depression, and represented the largest U.S. social insurance program at the time. While Social Security has been shown to generally mitigate welfare losses during deep economic downturns (Peterman and Sommer (forthcoming)), a large quantitative macro literature largely finds that the current program reduces steady state welfare in general equilibrium models. The findings from these studies raise a question as to why the program—given its welfare costs in the steady state—was implemented in the first place. To this end, our paper uses a general equilibrium, heterogeneous-agents life cycle model to quantitatively examine the welfare effects of the Social Security program’s adoption on the original cohorts of agents who experienced it.<sup>1</sup> We ask three questions. First, what were the overall welfare effects on individuals who were alive at the program’s adoption? Second, who were the winners and losers from the program’s enactment? And third, what were the main channels through which the adoption of the original program affected welfare?

We examine these questions in three steps. First, we build a rich, heterogeneous agent, general equilibrium life cycle model with endogenous labor and retirement that matches the U.S. economy just before the Great Depression and the enactment of the original Social Security program. Second, we introduce two sudden and unexpected shocks—the Great Depression and the subsequent adoption of the original Social Security—and calculate the transition path to a new, post-Great Depression steady state with Social Security fully phased in. Third, along the transition path, we study the welfare of the original cohorts of agents who lived through the Great Depression and the subsequent enactment of Social Security, and compare it to the welfare of agents who experienced a counter-factual transition path where the Great Depression occurs but Social Security is not adopted.

We measure the welfare effects of the original Social Security in two distinct ways. First, we determine the likelihood of a welfare gain from the adoption of Social Security for the original cohorts. Second, we calculate the average size of the welfare gains for agents in these cohorts. In contrast to the standard steady state results, our quantitative experiments suggest that the original program benefited a vast majority of agents who were alive at the time of the program’s enactment, with the average welfare effect being large and the gains being widespread. In particular, we estimate that the original program benefited households alive at the time of the program’s adoption with a likelihood of almost 90 percent, and increased these original agents’ welfare by the equivalent of 5.7 percent of their expected future lifetime consumption. These welfare benefits were particularly large for working-age individuals near retirement and also for agents with relatively less savings.

Compared to the standard steady state results, we find that the transitional agents experienced a large welfare benefit from the program’s adoption, because the ratio of their benefits to contributions was much larger than it would have

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<sup>1</sup>We focus on the original Social Security program which we define as the original program that was introduced in 1937 and started providing benefits in 1940. See Section 5.2 for a description of the original law and subsequent amendments prior to 1940.

been if they lived their entire life in the steady state with Social Security. Qualitatively, similar results have been derived in simple two-period models dating back to Samuelson (1958) and Diamond (1965). For instance, Feldstein and Liebman (2002) show that in a two-period dynamically efficient economy with a capital stock the initial generations receive a consumption windfall from Social Security at the expense of future generations.<sup>2</sup> Quantitatively, we find that similar mechanisms lead to large welfare gains for transitional agents in our rigorous model. For example, in our model, a transitional agent who was 60 years old at the time of the adoption of the program would face a lifetime payroll tax burden only 5 percent as big as the burden of an agent who lived their whole lifetime with Social Security, but would be entitled to a social Security benefit that was almost 60 percent as large as if he lived with the program his whole life.

The original cohorts contributed relatively less into the Social Security system for two primary reasons. First, the payroll tax rates were introduced at a low level and gradually scaled up over a number of years. Second, the original cohorts did not start paying into the system until the program was adopted, part way through their life. In contrast, the benefits were fully adopted immediately, resulting in total Social Security benefits that were considerably more generous relative to the contributions for these original cohorts. Moreover, the standard negative general equilibrium welfare effect of Social Security associated with capital crowd-out were also smaller during the transition than in the steady state because it took many periods for agents to adjust their savings levels in response to the program's adoption. Thus, along the transition, the general equilibrium effect merely muted the overall welfare gain from the program's adoption for the original cohorts.

Interestingly, and perhaps counter to simple intuition, we find that adopting the program during the Great Depression in fact tapered the welfare benefits from the program for the original cohorts. At first blush, one might be tempted to think that the Great Depression could have bolstered the welfare gains because the insurance from the Social Security benefits would be more valuable during the Great Depression when large amounts of wealth and income were lost. On the other hand, imposing a payroll tax on agents during the Great Depression when agents suffered from tighter budget constraints due to the adverse shock could lower the welfare gains from the program's adoption. On balance, we find that this latter channel dominates because most agents who were eligible for Social Security did not receive Social Security benefits for many years after the Great Depression, but had to start funding the system immediately, at the time when economic conditions were still depressed.

This paper is related to three strands of the existing literature. The first strand measures the long-run welfare effects of Social Security. These works generally weigh the relative benefit from Social Security providing partial insurance for risks for which no market option exists against the welfare costs of the distortions to an individual's incentives to work and save that the program imposes. Specifically, these studies examine the benefit from (i) providing intra-generational insurance for idiosyncratic risk from earnings and mortality (e.g., Hubbard and Judd (1987), Hubbard (1988), Imrohoroglu et al. (1995), Storesletten et al. (1998), Huggett and Ventura (1999), Imrohoroglu et al. (2003), Huggett and Parra (2010), and Imrohoroglu and Kitao (2012)), (ii) inter-generational insurance for aggregate risk

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<sup>2</sup>In this economy that is operating at a first-best equilibrium, the present value of the consumption losses of all current and future working generations is equal to the windfall consumption that the initial retirees receive. In contrast, the future generations lose because the implicit rate of return on payroll taxes is lower than the return agents would earn by investing those funds in the capital stock.

(Krueger and Kubler (2006)), or (iii) both (Harenberg and Ludwig (forthcoming)).<sup>3,4</sup> With a few exceptions, these studies generally find that Social Security is not welfare improving once general equilibrium effects of capital crowd-out are considered.<sup>5</sup> Similar to these papers, we quantify the welfare consequences of Social Security. However, this study is different in that it focuses on the welfare implications of Social Security over the transitional period after the program is adopted, as opposed to focusing on steady state effects once the program is well established.

The second, related strand of literature extends the steady state analysis with a study of transitional welfare after Social Security is either adopted, eliminated, or reformed (e.g., Auerbach and Kotlikoff (1987), Conesa and Krueger (1999), Cooley and Soares (1999), Krueger and Kubler (2006), Fuster et al. (2007), Olovsson (2010), Hong and Rìos-Rull (2007) and Kitao (2014)).<sup>6</sup> The three papers most closely related to our study are Auerbach and Kotlikoff (1987), Krueger and Kubler (2006), and Cooley and Soares (1999). The first two papers find that although a general Social Security program reduces steady state welfare, adopting the program can increase welfare for cohorts alive at the time of the program's introduction.<sup>7</sup> Further, Cooley and Soares (1999) show that adopting a generic Social Security system can emerge as a feasible political outcome in a general equilibrium rational-expectations model, because the median agent at the time of adoption benefits at the expense of the younger living agents and future cohorts.<sup>8</sup> Our paper contributes to this line of work by focusing in detail on the adoption of the original program in a historically consistent model that allows us to quantify how the law and economic background interacted in affecting welfare. Moreover, unlike these previous studies, our model incorporates idiosyncratic risk, thereby allowing us to assess how the welfare effects from the adoption of the program differed not only between cohorts but also between agents within the same cohort.

Another study in the strand of literature that focuses on Social Security and transitional welfare is Peterman and Sommer (forthcoming), which—similar to this paper—explores the interaction of Social Security and a large-scale recession. However, the questions and conclusions between these two papers differ in substantive ways. Specifically, Peterman and Sommer (forthcoming) shows that even though the current Social Security program is generally welfare-reducing, the established program can meaningfully mitigate welfare losses from large economic downturns. In contrast, this paper quantifies the implications of the introduction of the original Social Security program on welfare of agents who experience its adoption. We conclude that the introduction of the original Social Security program was gen-

<sup>3</sup>In studies with aggregate risk, there is no longer a deterministic steady state since different realizations of the aggregate shock will affect the economy. Thus, these studies either provide the range of welfare effects across different realizations of the potential paths for the aggregate shock, or the welfare effects under a particular sequence of shocks.

<sup>4</sup>Huggett and Ventura (1999), Huggett and Parra (2010), and Imrohoroglu and Kitao (2012) are examples of studies that considered welfare effects of reforms to the current program.

<sup>5</sup>Two notable exceptions are Imrohoroglu et al. (2003) and Harenberg and Ludwig (forthcoming). Imrohoroglu et al. (2003) shows that when preferences are time-inconsistent then the benefits of Social Security can outweigh the costs. Harenberg and Ludwig (forthcoming) find that Social Security can be welfare improving when both idiosyncratic and aggregate risks are present, but they generally consider a program that is quite small by historical standards.

<sup>6</sup>Instead of studying the adoption or elimination of the program, Olovsson (2010) and Kitao (2014) consider a transition to a reformed system, so the welfare consequences and transitional dynamics from these studies are not as comparable to our exercise.

<sup>7</sup>The studies which have examined a repeal or a reform that reduces the size of the existing Social Security system frequently find transitional welfare losses for the existing generations, even when the repeal or reform is welfare-improving in the long run. One notable exception is Fuster et al. (2007) who quantify the welfare effects of the US Social Security system in a two-sided altruism framework and compute the transitional dynamics implied by different reforms that eliminate social security. That paper concludes that a reform that finances the existing social security claims with debt and consumption taxes would benefit most individuals alive at the moment of the reform.

<sup>8</sup>Moreover, in their framework, Social Security can be subsequently sustained as a political equilibrium because the median worker in later generations, treating their past contributions as a sunk cost, would still find it in their interest to preserve the system until retirement, even though they would be better off living their whole life in an economy without Social Security.

erally welfare improving for these agents, but that introducing it during a severe recession reduced these welfare gains relative to a counter-factual implementation time line wherein the program is enacted in normal economic conditions.

Finally, our study is related to empirical literature that measures the average internal rate of return (ROR) of Social Security. This rate equalizes the present discounted value of the total average taxes paid and the average benefit payments for a given birth cohort. Consistent with our paper, these studies find the ROR from Social Security were the largest for cohorts already alive at the time when the program was adopted (see, for example, Leimer (1994), Leimer (2007), or Murphy and Welch (1998)). There are several differences between our paper and the ROR calculations. First, the ROR examines the average effect on each cohort, as opposed to the distribution of the effects within a cohort. Second, the ROR is not a utility-based measure. Since the ROR strictly captures the extent to which each cohort has received or can be expected to receive more or less resources from Social Security than they contributed, it does not reflect all the welfare effects of Social Security, such as welfare benefits from insurance and welfare cost from the payroll tax exacerbating liquidity constraints.<sup>9</sup>

This paper is organized as follows. Section 2 introduces the computational model. Section 3 presents the dynamic programming problem. Section 4 describes parametrization of the steady state economies and compares the initial steady state without Social Security to the available pre-Depression U.S. data. In Section 5, consistent with historical experience, we parametrize the economic shocks associated with the Great Depression and the phase-in of the original Social Security program, and—where possible—compare the simulated transitional path to the historical data. In Section 6, we describe our computational experiment, define our welfare measure, present our welfare findings, and provide some sensitivity analyses. Section 7 concludes.

## 2. MODEL

Our framework is a general equilibrium, life cycle economy with overlapping generations of heterogeneous agents, uniquely built and calibrated to quantify the welfare effects of the adoption of the original Social Security program on agents who were alive at the time of the program's adoption. The initial steady state is calibrated to the U.S. economy prior to the Great Depression in which no Social Security exists. We then introduce the Great Depression, after which the economy transitions on a perfect foresight path. However, this path is altered by a second unexpected shock, the introduction of Social Security. Thus, the final steady state represents the U.S. economy after a transition through the Great Depression and the adoption of Social Security in accordance with the historical law.

### 2.1. *Demographics*

Time is assumed to be discrete, and the model period is equal to one year. Agents, indexed by age  $j$ , enter the model when they start working ( $j = 1$ ), and live up to a maximum possible age of  $j = J$ . Thus, in each period, the economy is populated by  $J$  overlapping generations of individuals of ages  $j = 1, \dots, J$ . The size of each new cohort

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<sup>9</sup>The ROR and welfare frameworks also use different bases of comparison. The welfare calculation compares the welfare in an economy where Social Security exists to welfare in a counter-factual economy without Social Security, and consequently also incorporates the differential general equilibrium effects between these environments. In contrast, the ROR simply compares Social Security taxes paid versus benefits received in an environment where the program exists, without accounting for the general equilibrium effects.

grows at a constant rate  $n$ . Lifetime length is uncertain, with mortality risk rising over the lifetime. The conditional survival probability from age  $j$  to age  $j + 1$  is denoted  $\Psi_j$  where  $\Psi_J = 0$ . Annuity markets do not exist to insure life-span uncertainty and agents are assumed to have no bequest motive. In the spirit of Conesa et al. (2009), accidental bequests, which arise from the presence of mortality risk, are distributed equally amongst the living in the form of transfers  $Tr$ . Agents endogenously choose the age  $R$  at which they retire. The binary decision to retire (i.e.,  $I = \{0, 1\}$  where  $I = 1$  denotes the event of retirement) is considered irreversible and is restricted to be within the age range of  $[\underline{R}, \bar{R}]$ .

## 2.2. Endowments, Unemployment, Preferences and Market Structure

In each period, an agent is endowed with time that can be used for leisure or market work. An agent's labor earnings are given by  $y = w\omega h(1 - D)$ , where  $w$  represents the wage rate per efficiency unit of labor;  $h$  is the fraction of the available time endowment spent on labor market activities;  $D$  is the fraction of the time endowment in each period that the agent forfeits to unemployment; and  $\omega_t$  is the idiosyncratic labor productivity.

We assume that idiosyncratic labor productivity follows the process:  $\log \omega = \theta_j + \alpha_0 + v$ . In this specification,  $\theta_j$  governs the deterministic age-profile of productivity; and  $\alpha_0 \sim NID(0, \sigma_\alpha^2)$  is an individual-specific fixed ability type that is observed when an agent enters the economy and stays fixed for an agent over the life cycle. Finally,  $v$  is a persistent shock, received each period, which follows a first-order auto-regressive process:  $v = \rho v_{-1} + \zeta$ , with  $\zeta \sim NID(0, \sigma_\zeta^2)$  and  $v_1 = 0$ .

Our modeling approach to unemployment shocks is derived from Kaplan (2012) who introduces a role for unemployment shocks into an annual model. Since unemployment spells are typically shorter than one year, they are introduced as exogenous reductions in the available annual time endowment, with the exogenous independent and identically distributed unemployment shock,  $D$ , discretized to two values: zero and  $d \in (0, 1]$ . The unemployment shock arrives with a probability  $p^U$ . Conversely, the probability of not experiencing an unemployment spell within a period is  $(1 - p^U)$ . When an unemployment spell hits the agent loses the option to work during  $d$  percent of their time endowment. Since  $h$  is the fraction of available time spent on labor market activities, the total labor supplied by an agent is equal to  $(1 - D)h$ . Thus, the resulting model for an agent prior to retirement is a hybrid between one that treats labor supply as a choice (the frictionless intensive margin) and a constraint which is introduced by the unemployment shock.

This approach of introducing unemployment naturally lends itself to our setting, because an annual model is considerably more tractable than a higher frequency model when calculating the transitional path over 100 years. However, one downside is that the approach limits us from introducing an unemployment spell that lasts longer than one year.<sup>10</sup> Although this assumption seems consistent with the limited historical data outside of the Great Depression, there is evidence that during parts of the Great Depression the average duration of an unemployment spell increased to over a year.<sup>11</sup> Providing some comfort that this limitation does not have large consequences for our results, in Section 6.4

<sup>10</sup>Allowing the duration to last more than one year in the model would require an increase in the size of the state space vector to include an indicator of whether agents were unemployed in the previous period.

<sup>11</sup>For example, Palmer (1937) reports average duration of unemployment spells increased from approximately 4 month in 1929 to 2 years during

we show that the welfare effects of Social Security are fairly insensitive to increases in the average duration of an unemployment spell over the Great Depression.

An agent's preferences over the life cycle are governed by the time-separable utility function:

$$E_0 \sum_{j=0}^J \beta^j (u(c) + v(h, D)),$$

where  $c$  is the stream of consumption; and as noted before,  $h$  is the percent of the available time endowment an agent chooses to work, while  $D$  is the percent of the time endowment that is unavailable for work due to an unemployment spell.  $\beta$  is the discount factor. Expectations are taken with respect to the life-span uncertainty, the idiosyncratic labor productivity risk, and the unemployment risk.

Agents can hold savings in the form of assets,  $a \geq 0$ . Agents choose to save for two reasons. First, they save to partially insure against idiosyncratic labor productivity, unemployment, and mortality risks. Moreover, they save in order to fund their post-retirement consumption. Once Social Security is adopted, the program provides another source of funds for this consumption. Agents can hold savings in the form of assets,  $a \geq 0$ . Agents choose to save for two reasons. First, they save to partially insure against idiosyncratic labor productivity, unemployment, and mortality risks. Moreover, they save in order to fund their post-retirement consumption. Once Social Security is adopted, the program provides another source of funds for this consumption.

### 2.3. Technology

Firms are perfectly competitive with constant returns to scale production technology. Thus, we use a representative firm with a Cobb-Douglas production function  $Y = F(A, K, N) = AK^\zeta N^{1-\zeta}$ , where  $A$ ,  $K$ ,  $N$ , and  $\zeta$  are aggregate Total Factor Productivity (TFP), capital, labor, and the capital share of output, respectively. Capital depreciates at a constant rate  $\delta \in (0, 1)$ . The firm rents capital and hires labor from agents in competitive markets, where factor prices  $r$  and  $w$  are equated to their marginal productivity. The aggregate resource constraint is:  $C + K' - (1 - \delta)K + G \leq AK^\zeta N^{1-\zeta}$  where, in addition to the above described variables,  $C$  and  $G$  represent aggregate household and government consumption, respectively.

### 2.4. Government Policy

The government distributes accidental bequests to the living in a form of lump-sum transfers,  $Tr$ , and consumes in an unproductive sector.<sup>12</sup> Government consumption,  $G$ , is exogenously determined, and is modeled as proportional to the total output in the steady state economy, so that  $G = \phi Y$ . The level of government consumption is determined in the steady state without Social Security and is held constant throughout the transition. Once Social Security is enacted, the government additionally collects a proportional Social Security tax,  $\tau^{ss}$ , on pre-tax labor income of working-age individuals (up to an allowable taxable maximum  $\bar{y}$ ) to finance Social Security payments,  $b^{ss}$ , for retired workers.

the mid 1930s in the Philadelphia labor market.

<sup>12</sup>By the timing convention, agents realize at the beginning of the period whether they die. Subsequently, the transfers are received at the beginning of the period before agent's idiosyncratic labor productivity status is revealed.

The government taxes income according to a schedule  $T(\tilde{y})$  in order to raise revenue to finance its consumption in the unproductive sector. The taxable income,  $\tilde{y}$ , is defined as:  $\tilde{y} = y + r(Tr + a) - 0.5\tau^{ss} \min\{y, \bar{y}\}$ . The part of the pre-tax labor income ( $y$ ) that is accounted for by the employer's contributions to Social Security,  $(0.5\tau^{ss} \min\{y, \bar{y}\})$ , is not taxable. In the benchmark steady state with no Social Security,  $\tau^{ss}$  is set to zero.

Similar to the current system, the original Social Security benefits were calculated as an increasing, concave, piecewise-linear function of worker's average level of lifetime labor earnings. However, the original program was considerably less progressive, with the benefits formula being governed by a single bend point and two marginal replacement rates. Unlike the current program, the original Social Security benefits were also adjusted for the number of years in which an individual contributed payroll taxes, and the benefits were disbursed only after an agent reached the normal retirement age (NRA) of 65.<sup>13</sup>

In the final steady state with Social Security, Social Security benefits are calculated in three steps. First, we compute each worker's average level of labor earnings over the working life cycle,  $x_R$ . At every age, the total accumulated earnings follow the law of motion:

$$(2.1) \quad x_{j+1} = \frac{\min\{y_j, \bar{y}\} + (j-1)x_j}{j},$$

where  $x_j$  is the accounting variable capturing the equally-weighted average of earnings before the endogenously chosen retirement age  $R$ ; and  $\bar{y}$  is the maximum allowable level of labor earnings subject to the Social Security tax that corresponds to the benefit-contribution cap.<sup>14</sup> Second, for each retiree, the pre-adjustment Social Security benefit,  $b_{base}^{ss}$ , is calculated using a convex, piecewise-linear function of average past earnings observed at retirement age,  $x_R$ . The function allows the marginal replacement rate to vary over three levels of taxable income:

$$(2.2) \quad \begin{aligned} &\tau_{r1} \quad \text{for } 0 \leq x_R < b_1 \\ &\tau_{r2} \quad \text{for } b_1 \leq x_R < b_2 \\ &0 \quad \text{for } x_R \geq b_2. \end{aligned}$$

The parameter  $b_1$  is the first bend point; the parameter  $b_2$  is the benefit-contribution cut-off point ( $b_2 = \bar{y}$ ); and the parameters  $\{\tau_{r1}, \tau_{r2}\}$  represent the marginal replacement rates for the pre-adjustment Social Security benefit.

Finally, an adjustment is made to the benefits to account for the number of years of payroll tax contributions. In particular, for each year that agents pay payroll taxes, their benefits are scaled up by the equivalent of one percent. As a result, the total Social Security benefit,  $b^{ss}$ , received by the retiree is defined as:

$$(2.3) \quad b^{ss} = b_{base}^{ss} \times \left(1 + \frac{R}{100}\right).$$

<sup>13</sup>The current system has two bend points and three marginal replacement rates. Moreover, it allows individuals to claim Social Security benefits prior to reaching their NRA. Finally, there are no adjustments to the Social Security benefits for the number of years worked; rather, only the top thirty years of income are considered.

<sup>14</sup>If an agent chooses to retire prior to the NRA, then their average earnings for non-working years prior to reaching the NRA are populated with zero. Additionally, if an agent chooses to work past the NRA then the additional years worked past the NRA are factored into their lifetime average earnings from which the ultimate Social Security benefits are computed.



However, the benefit is subject to a minimum and a maximum, such that  $b^{ss} \in [b_{min}^{ss}, b_{max}^{ss}]$ .

Along the transitional period after Social Security is introduced, equations 2.1 and 2.3 are altered to reflect the historical law during the program's phase-in; we describe these adjustments in Section 5.2.

### 3. DYNAMIC PROGRAM

For expositional convenience, this section introduces the dynamic program of an individual who enters the economy in the final steady state with Social Security. We present two separate dynamic programming problems: one for an agent who was not yet retired in the previous period, and one for an agent who was retired. In the initial steady state without Social Security, the dynamic programming problem is simplified by setting  $\tau^{ss}$  and  $b^{ss}$  to zero. Appendix A provides a formal definition of the market equilibrium and the balanced growth path.

Starting with retired agents, these agents are no longer affected by labor productivity or unemployment shocks because they no longer work. Thus, a retired agent is indexed by type  $(a, b^{ss}, j)$  and solves the dynamic program:

$$(3.1) \quad V_t^R(a, b^{ss}, j) = \max_{c, a'} u(c) + \beta \Psi_j EV^{R'}(a', b^{ss}, j+1),$$

subject to

$$(3.2) \quad c + a' = (1+r)(Tr+a) + b^{ss} - T(\tilde{y}),$$

by choosing consumption,  $c$ , and savings,  $a'$ . Retirees earn interest income  $r(Tr+a)$  on the transfer,  $Tr$ , and their existing asset holdings,  $a$ . These agents who are older than the NRA also receive the constant per-period Social Security payment,  $b^{ss}$ , once the program is implemented.

An agent who is younger than the minimum retirement age,  $\underline{R}$ , cannot retire in the current period. These agents are indexed by type  $(a, x, \alpha, v, j, D)$  and solve the dynamic program:

$$(3.3) \quad V(a, x, \alpha, v, j, D) = \max_{c, a', h} (u(c) + v(h, D)) + \beta \Psi_j EV'(a', x', \alpha, v', j+1, D')$$

subject to

$$(3.4) \quad c + a' = (1+r)(Tr+a) + y - T(\tilde{y}) - \tau^{ss} \min\{y, \bar{y}\} \text{ and } 0 \leq h \leq 1$$

by choosing consumption,  $c > 0$ , savings,  $a' \geq 0$ , the fraction of available time endowment spent on working,  $h$ . These agents also earn interest income  $r(Tr+a)$  on the lump-sum transfer from accidental bequests,  $Tr$ , and on asset holdings,  $a$ .  $y$  represents the pre-tax labor income of the working agents and  $\tilde{y}$  defines the taxable income on which the income tax,  $T$ , is paid.  $D \in \{0, d\}$  is the state variable for the fraction of the period an agent is exogenously unemployed. The Social Security tax rate,  $\tau^{ss}$ , is applied to the pre-tax labor income,  $y$ , up to an allowable taxable maximum,  $\bar{y}$ , and  $b^{ss}$  denotes the individual-specific constant Social Security benefit that is received by retired agents every period after reaching the NRA.

Finally an agent who has not yet retired and is between the ages of  $\underline{R}$  and  $\bar{R}$  has a choice of whether to retire in the current period. These agents are also indexed by type  $(a, x, \alpha, v, j, D)$  and solve the dynamic program:

$$(3.5) \quad W(a, x, \alpha, v, j, D) = \max_{c, a', h, I \in \{0, 1\}} (u(c) + v(h, D)) + \begin{cases} \beta \Psi_j EV'(a', x', \alpha, v', j+1, D') & \text{if } I = 0, \\ \beta \Psi_j EV^R(a', x', j+1) & \text{if } I = 1, \end{cases}$$

subject to

$$(3.6) \quad \begin{aligned} c + a' &= (1+r)(Tr+a) + y - T(\bar{y}) - \tau^{ss} \min\{y, \bar{y}\} \text{ and } 0 \leq h \leq 1 & \text{if } I = 0, \\ c + a' &= (1+r)(Tr+a) - T(\bar{y}) + b^{ss} & \text{if } I = 1. \end{aligned}$$

where  $I$  indicates whether an agent chooses to permanently retire in the current period.

#### 4. STEADY STATE CALIBRATION

We begin by calibrating the initial steady state that excludes Social Security. The parameters in the model are calibrated such that the model reproduces key moments of the U.S. data. To the extent that reliable data are available, we use historical data prior to the Great Depression and the subsequent adoption of the original Social Security program to parametrize the model. After calibrating the benchmark economy without Social Security, we parametrize the original Social Security program and compute the final steady state while keeping all other non-Social Security parameters constant. Table I summarizes the parameters used to calibrate the initial steady state. Table II parametrizes the original Social Security program.

##### 4.1. *Demographics, Endowments, Unemployment Risk and Preferences*

There are 74 overlapping generations of individuals of real-life ages ranging between 20 (i.e.,  $j = 1$ ) to 93 (i.e.,  $J = 74$ ). The population growth rate,  $n$ , is set to 1.6 percent to match the average U.S. annual population growth (reported by the Census Bureau) from 1920 through 1928. The conditional survival probabilities,  $\Psi_j$ , are derived from the U.S. life tables for the 1930s (Bell and Miller (2002)). To increase the computational tractability of the model, the minimum and maximum ages at which an agent is allowed to retire ( $\underline{R}$  and  $\bar{R}$ ) in the model are set at a real world age of 60 (i.e.,  $j = 41$ ) and 85 (i.e.,  $j = 66$ ), respectively.<sup>15</sup>

Ideally, to calibrate the wage process, we would rely on panel data on wages. However, such historical data are not available. Given the lack of data, we follow Conesa et al. (2009) in calibrating the process for the labor productivity,  $\omega$ , based on cross-sectional wage data from the 1940 Census.<sup>16</sup> We restrict the estimation sample to male household heads who were between ages 20 and 64, worked at least five weeks, and worked more than 1,248 hours over the year. To pin down the deterministic age-specific productivity profile, we regress natural log of average wages on a quadratic

<sup>15</sup>Constraining the binary retirement decisions to 25 years reduces number of periods in which such decisions are made, thereby reducing the state space. That said, disallowing agents from retiring prior to age 60 in the model does not seem to be inconsistent with the data, as less than 10 percent of all male household heads were reported out of labor force in either the 1920 or the 1930 Census.

<sup>16</sup>Ideally, the productivity process would be calibrated from data prior to the Great Depression and the implementation of Social Security. Unfortunately, to the best of our knowledge, such data are not readily available prior to 1940. To reduce the effects of the adoption of Social Security in 1940 on our estimates, our analysis focuses on observations for individuals who were younger than the NRA in 1940. However, we are unable to control for the effects that the adoption of Social Security might have had on labor supply and wage dynamics of younger individuals.

TABLE I  
PARAMETERS USED TO PARAMETRIZE THE INITIAL STEADY STATE

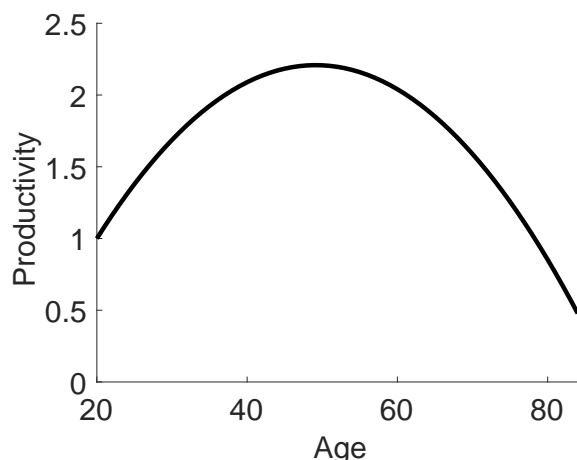
Exogenous Parameters	Value	Source of the Parameter Value	
Normal Retirement Age: $\overline{NRA}$	65	By assumption, based on the U.S. SS Program	
Minimum Retirement Age: $\underline{R}$	60	By assumption	
Maximum Retirement Age: $\overline{R}$	85	By assumption	
Maximum Age: $J$	93	By assumption	
Age-Specific Survival Probabilities: $\Psi_j$		Estimates from the 1930 SSA Period Table	
Population Growth: $n$	1.6%	Estimates from the 1928 Census	
Capital Share Parameter: $\zeta$	.32	Based on a 1929-30 estimate from Piketty and Saez (2003)	
Total Factor Productivity: $A$	1	Normalized	
Risk Aversion: $\gamma$	2	Based on Conesa et al. (2009)	
Frisch Elasticity: $\sigma$	0.5	Based on a host of studies; see footnote 19	
Disutility of Unemployment: $\xi$	0.00	Based on a estimate in Kaplan (2012) which uses 1968-2005 data	
Persistent Shock: $\sigma_v^2$	0.007	Derived to match age-profile of variance of wages in the 1940 Census	
Persistent Shock: $\rho$	0.990	Derived to match age-profile of variance of wages in the 1940 Census	
Permanent Shock: $\sigma_\alpha^2$	0.437	Derived to match age-profile of variance of wages in the 1940 Census	
Unemployment Rate: $p^u$	4.1%	Based on the NBER series for periods 1945-50	
Unemployment Duration: $d$	0.30	Based on the Philadelphia Labor Survey (1929)	
Government Spending Share: $\phi$	2.8%	Based on BEA data for 1929-30	
Endogenous Parameters	Value	Target	Data Source (Year)
<b>Determined through Calibration:</b>			
Capital Depreciation Rate: $\delta$	6.90%	$\frac{I}{Y} = 25.6\%$	BEA: 1929-30
Conditional Discount: $\beta$	0.993	$\frac{K}{Y} = 3.0$	BEA: 1930-31
Disutility to Labor: $\chi_1$	72.9	Avg. $h_j = .282$	Census: 1940
Fixed Cost to Working: $\chi_2$	0.495	14.3% retired at age 65	Census: 1930
Tax Exemption Parameter: $\Upsilon_1$	.57	50% of tax filers paid no taxes	Tax Foundation: 1920-40
<b>Determined through Market Clearing:</b>			
Income Tax Rate: $\Upsilon_0$	0.128	Balanced Government Budget	

Notes: Ages are denoted in real world ages as opposed to model ages. Agents enter the model at age 20.

TABLE II  
ADDITIONAL PARAMETERS USED TO PARAMETRIZE THE FINAL STEADY STATE

Exogenous Parameters	Value	Data Source
Marginal Replacement Rate: $\tau_{r1}$	40%	U.S. SS Program
Marginal Replacement Rate: $\tau_{r2}$	10%	U.S. SS Program
Bend Point: $b_1$	.57 x Avg Earnings	U.S. SS Program & NBER
Social Security Benefit-Contribution Cut-off: $\bar{y}^{***}$	2.84 x Avg Earnings	U.S. SS Program & NBER
Minimum Social Security Benefit: $b_{min}^{SS}$	0.11 x Avg Earnings	U.S. SS Program & NBER
Maximum Social Security Benefit: $b_{max}^{SS}$	0.97 x Avg Earnings	U.S. SS Program & NBER
Endogenous Parameters (Determined through Market Clearing)	Value	Target
Payroll Tax: $\tau^{SS}$	4.46%	Market Clearing

FIGURE 1.— Deterministic Age Profile of Productivity



Note: The data are from the 1940 Census. This deterministic age profile is calculated from a regression of average hourly earnings on a quadratic polynomial and normalized to 1 at age 20.

polynomial of age, and normalize the exponential transformation of this profile to one at the real world age of 20. This exponential transformation is shown in Figure 1.

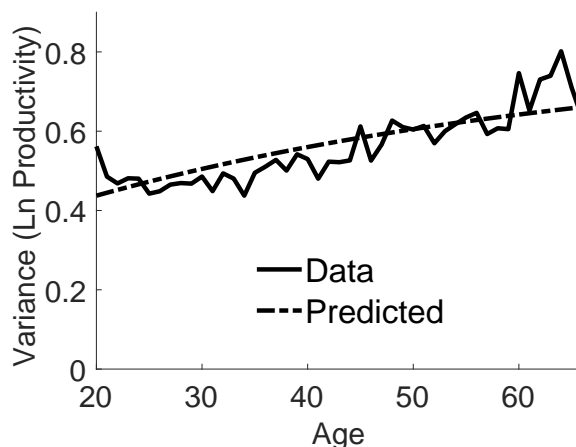
Having calibrated the deterministic age-profile, we next use the age-specific variance of the natural log of average annual hourly earnings by age (shown in Figure 2) to infer the parameter values for the permanent and persistent shocks to the individuals' productivity. First, we set the variance of the permanent shock,  $\sigma_{\alpha}^2$ , to 0.437 in order to match the minimum variance of the natural log of average annual hourly earnings between ages 20 and 30 in the data. Second, turning to the persistent productivity shock, we set  $\rho = 0.990$  to match the linear growth of the variance in average annual hourly earnings over the life cycle, depicted by the solid line in Figure 2. Finally, we set  $\sigma_{\nu}^2$  so that its calibrated value minimizes the sum of squared percentage deviations between the empirical and simulated variance of average annual hourly earnings at each age (plotted in Figure 2). In order to solve the model, we discretize the permanent and persistent shock with two and five states, respectively.<sup>17</sup>

To calibrate the duration of the unemployment shock, we rely on the best available data, which is the Philadelphia Labor Survey (Palmer (1937)). The survey focuses on the Philadelphia labor market from 1929 to 1937. Consistent with the 1929 data, we calibrate the unemployment shock  $D \in \{0, d = 0.3\}$ , so that each agent hit by an unemployment spell spends thirty percent of the year (or approximately 4 months) being involuntary unemployed. The unemployed agent can spend the remaining 70 percent of the period on work and leisure. Turning to the probability of an unemployment shock, we set  $p^U$  to match the national average unemployment rate of 4.1 percent over the period 1945-1950 in the NBER unemployment series.<sup>18</sup> In all, agents have a 4.1 percent chance of being unemployed at any given time, with the unemployment spell lasting for 30 percent of the year.

<sup>17</sup>Given the highly persistent process, we use the Rouwenhorst method to discretize the productivity process.

<sup>18</sup>The NBER series compiles estimates from several different sources; for details, see <http://www.nber.org/databases/macroeconomy/contents/chapter08.html>. We use the NBER estimates from 1945-1950, because a sufficiently long time series of estimates of the unemployment rate prior to the Great Depression do not exist. However, the average estimate for the 1945-1950 period is fairly close to some of the available pre-Great Depression estimates (i.e. see Darby (1976) and Lebergott (1964)), which report unemployment of about 3 percent in 1929.

FIGURE 2.— Variance of the Log of Average Annual Hourly Earnings



Note: The data are from the 1940 census. The variance from the data is calculated as the variance in average hourly earnings for each cohort.

In the spirit of Kaplan (2012), household preferences are modeled as:

$$u(c) + v(h, D) = \frac{c^{1-\gamma}}{1-\gamma} - \chi_1 \frac{((1-D)\xi h)^{1+\frac{1}{\sigma}}}{1+\frac{1}{\sigma}} - \chi_2(1-I),$$

with the binary indicator  $I = 1$  denoting whether an agent is retired in the current period. The constant relative risk aversion preferences over consumption are characterized by the risk aversion coefficient,  $\gamma$ , which determines an agent's desire to smooth consumption across time and states. The existing estimates of  $\gamma$  (though generally based on more recent data) typically range between 1 and 3. Given the lack of historical estimates, we set  $\gamma = 2$ , which is consistent with Conesa et al. (2009). The parameter  $\sigma$  represents the Frisch labor supply elasticity. Past microeconomic studies estimate the Frisch elasticity to be between 0 and 0.5.<sup>19</sup> However, more recent research shows that these estimates may be biased downward.<sup>20</sup> We calibrate  $\sigma$  to 0.5—the upper range of the available estimates.

The parameter  $\xi$  determines the flow of disutility an agent receives during unemployment spells. Recall from our discussion in Section 2.2 that  $(1-D)h$  determines the total labor supplied by an agent. That said, note that it is the expression  $(1-D)\xi h$  which enters the agent's utility. Thus, when  $\xi = 1$  the two terms are equal and the agent derives no disutility during unemployment spells (i.e., time spent unemployed is treated equivalent to leisure). However, when  $\xi < 1$  then  $(1-D)h \leq (1-D)\xi h$ , meaning that there is disutility associated with time spent in unemployment. In accordance with estimates in Kaplan (2012), we set  $\xi = 0$ , which implies that during the part of the period when an agent is unemployed they derive disutility at the same rate as if they were working. Because there is disutility associated with time spent in unemployment, an agent will tend to lower the amount of total labor supplied when they experience an unemployment shock.<sup>21</sup>

The remaining parameters are calibrated endogenously to match external data moments. Specifically, the discount

<sup>19</sup>See, for example, Kaplan (2012), Altonji (1986), Peterman (2016), MaCurdy (1981), Domeij and Floden (2006) or Browning et al. (1999).

<sup>20</sup>See Imai and Keane (2004), Domeij and Floden (2006), Pistaferri (2003), Chetty (2012), and Contreras and Sinclair (2008).

<sup>21</sup>Kaplan (2012) estimates  $\xi = -0.08$  but not statistically different from zero. Kaplan (2012) estimates  $\xi$ , along with  $\gamma$  and  $\sigma$ , using the PSID data. The estimates for  $\gamma$  and  $\sigma$  are also in line with the calibration values we use in the model.

factor,  $\beta$ , is calibrated to 0.993 to endogenously match the U.S. capital-to-output ratio of 3.0 reported by the Bureau of Economic Analysis for the period 1929-1931.<sup>22</sup> Moreover, the scaling constant  $\chi_1$  is calibrated such that agents spend, on average, 28.2 percent of their time endowment working prior to reaching the NRA, corresponding to the 1940 Census in which male household heads worked on average 1,760 hours per annum.<sup>23</sup> Finally, consistent with the 1930 Census, the fixed cost of working,  $\chi_2$ , is calibrated so that 14.3 percent of male heads of household retire by the NRA.<sup>24</sup> Without  $\chi_2$ , two factors jointly affect agents' retirement decisions: (i) declining productivity at the end of the working lifetime (shown in Figure 1), and (ii) the Frisch elasticity,  $\sigma$ . With the empirically estimated Frisch elasticity of 0.5, the decline in wages is too shallow to induce many agents to retire by the NRA. Incorporating  $\chi_2$  implies that an agent's disutility from working discontinuously drops when an agent retires, which allows the model to match the retirement decisions in the data.<sup>25</sup>

#### 4.2. Firm

The aggregate production function is Cobb-Douglas. The capital share parameter,  $\zeta = 0.32$ , is set to match the 1929-1930 average value drawn from Piketty and Saez (2003) (see their Figure 6). The depreciation rate is calibrated such that the investment to output ratio is 25.5 percent, as reported by the BEA in 1929 and 1930. TFP parameter,  $A$ , is normalized to unity in the baseline steady state, but varies along the transitional path in accordance with data (see Section 5).

#### 4.3. Government

Government spending in the unproductive sector,  $\phi$ , is set to 2.8 percent of GDP, consistent with the ratio of Federal Government expenditures to GDP reported by the BEA in 1929 and 1930. Turning to the income tax function, in the 1930s, the federal tax policy was much less progressive than the current system. In particular, a large fraction of taxable income was tax-exempt, and the rest of the income was subject to a fairly flat tax schedule with relatively low marginal rates.<sup>26</sup> Consequently, close to 50 percent of tax returns had zero or negative tax liability in the 1930s.<sup>27</sup> Thus, we model the stylized income tax policy as:

$$T(\tilde{y}_t; Y_0, Y_1) = Y_0 \max\{\tilde{y}_t - Y_1, 0\},$$

<sup>22</sup>Capital is calculated as the sum of private fixed assets and consumer durables reported by the Bureau of Economic Analysis. The values are not reported prior to 1929.

<sup>23</sup>Ideally hours would be calibrated to the data prior to the implementation of Social Security. However, hours data are not available from the Census until 1940. In order to get around the effects of Social Security on hours, we calibrate to hours worked for individuals who are too young to be eligible to collect Social Security benefits.

<sup>24</sup>Given that the Census data for this period does not directly report retirement status, individuals who are not in the labor force in the Census data are considered retired. This assumption seems reasonable since less than five percent of households under the age of 55 are reported as not in the labor force.

<sup>25</sup>We find that in the calibrated model the disutility from the fixed cost of working ( $\chi_2$ ) is the equivalent to approximately one-quarter of the utility from the average level of consumption. See Rogerson and Wallenius (2009) for further discussion of how including a non-convexity in the translation between time spent working (labor hours) and productive labor (labor services) can induce a working life in which after beginning work agents work continuously through retirement and the non-convexity can change the length of time an agent chooses to work.

<sup>26</sup>The first \$2,500 of income for married households and \$1,000 for single filers was tax-exempt. Moreover, the marginal tax rate for the part of the first \$4,000 of income that was not exempt was flat at four percent, and then increased only very gradually for higher income. These exemption levels and the limit on the first tax bracket were quite high compared to the mean individual income of \$1,054 in 1929 (calculated from the Macroeconomic historical data from the National Bureau of Economic Research).

<sup>27</sup>Source: Tax Foundation (<http://taxfoundation.org/article/federal-individual-income-tax-returns-zero-or-negative-tax-liability-1916-2010>)

where  $\Upsilon_0$  is the flat marginal tax rate and  $\Upsilon_1$  controls the level of the tax exemption.  $\Upsilon_1$  is calibrated so that 50 percent of tax filers do not pay any taxes in the initial steady state. Moreover, we calibrate  $\Upsilon_0$  such that the government budget constraint clears. We find that the marginal rate of 12.8 percent clears the government's budget, implying an average tax rate of 4 percent. This rate is generally consistent with the average historical income tax rates (defined as ratio of the total income to the total tax liability), which varied between 2.6 and 4.3 percent from 1923-1930 according to the Tax Policy Center.<sup>28</sup>

#### 4.4. *Social Security*

In the final steady state with Social Security, we set the NRA to 65 and set marginal replacement rates ( $\tau_{r1}, \tau_{r2}$ ) to their respective historical values of 0.4 and 0.1. Similarly, in the spirit of Huggett and Parra (2010), we set the bend point ( $b_1$ ), the maximum earnings ( $\bar{y}$ ), the minimum benefit ( $b_{min}^{ss}$ ), and the maximum benefit ( $b_{max}^{ss}$ ) so that they occur at 0.57, 2.84, 0.11, and 0.97 times mean earnings in the economy.<sup>29</sup> In the final steady state, we set  $\tau^{ss} = 0.045$ , so that the Social Security program's budget is balanced.<sup>30</sup>

#### 4.5. *A Comparison of the Baseline Steady State Economy to the U.S. Data*

As an external test of our benchmark steady state model, it is helpful to compare some of the endogenously generated moments summarizing households' retirement and savings decisions to the available historical data. Although limited by data, we are able to do two checks. Figure 3 plots the fraction of male household heads age 60+ who are not in the labor force in the data against the fraction of retired agents in the initial steady state without Social Security. Even though in the calibration we only directly target the fraction of retired households at age 65 (14.3 percent), the fraction of retired households endogenously generated by the model (the black dashed line) looks remarkably similar to the data (the black solid line) across most of the age range. The baseline model also generates a wealth to income ratio of 3.79, which is consistent with the estimate of the ratio of 3.79 for the ten years prior to the Great Depression from Saez and Zucman (2016).<sup>31</sup> Overall, the ability of the model to endogenously generate retirement and savings decisions that produce moments which match the pre-Depression data is encouraging.

A comparison of retirement and savings decisions in the final steady state to the U.S. data is complicated by the fact that the model economy takes approximately 50 periods to converge from the initial to the final steady state once it sets on a transitional path. Over this transitional period, Social Security has expanded significantly and also become more progressive. Moreover, there were a number of other additional changes to the U.S. fiscal policy, such as increases in income taxes, changes to income tax progressivity, and increases in the size of government spending. These post-adoption changes to Social Security and fiscal policy—which were largely unforeseen by agents at the time of the inception of Social Security—are excluded from our experiments by design, as the purpose of this analysis is

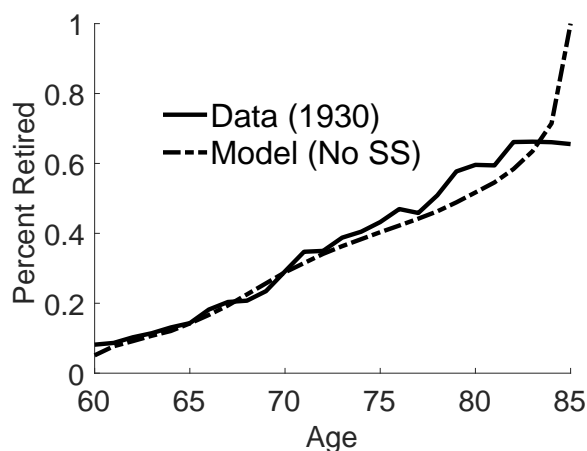
<sup>28</sup>See <http://www.taxpolicycenter.org/taxfacts/displayafact.cfm?DocID=564&Topic2id=30&Topic3id=39>

<sup>29</sup>See <http://www.nber.org/databases/macrophistory/contents/>.

<sup>30</sup>In reality, the actual rate hovered around a slightly higher level of about 5 percent over this period. However, some of this revenue was used to fund other parts of the Social Security program that were not related to the retirement benefits.

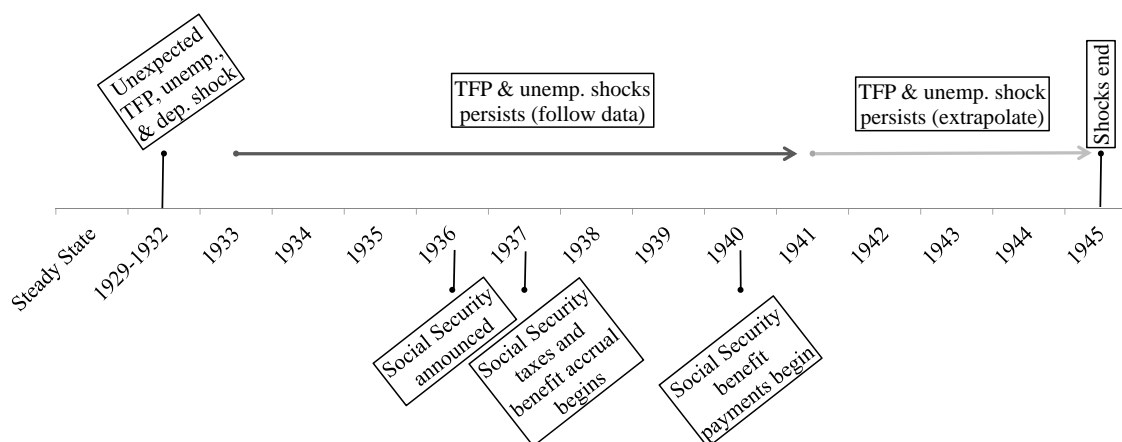
<sup>31</sup>It would be interesting to compare the age-profile of savings and consumption to the data. Unfortunately, data allowing such a comparison are not available.

FIGURE 3.— Percent Retired



Note: The data are from the 1930 Census. We limit the sample to males who are head of their household. Given that the Census data for this period does not directly report retirement status, in the data, individuals who are not in the labor force are considered to be retired. The model captures the percent of retired individuals in the steady state without Social Security.

FIGURE 4.— Time Line



to study the welfare effects of the enactment of Social Security as they were likely perceived by the original cohorts. Excluding the economic effects of these policy changes prevents us, however, from calibrating the final steady state to the historical U.S. data, as they are not comparable.

##### 5. CALIBRATION OF THE TRANSITIONAL PATH

Having parametrized the initial and final steady states, this section parametrizes (i) the economic shocks associated with the Great Depression and (ii) the phase-in of the original Social Security program. Both the Great Depression and the phase-in of Social Security are incorporated in the model consistent with the actual historical experience. Figure 4 outlines the time line of these events, which are subsequently discussed in Sections 5.1 and 5.2. Section 5.3 in turn compares the transitional path produced by the model to the evolution of the U.S. economy following the Great Depression and through the beginning of the World War II (WWII).



### 5.1. *The Great Depression*

We model the initial unexpected economic downturn associated with the Great Depression as one that affects the economy through three distinct channels: an adverse TFP shock, an adverse unemployment shock, and an adverse capital depreciation shock. We calibrate these shocks to match the total changes in the available empirical estimates of the TFP, unemployment rate, and capital stock between 1929 and 1932 (see time line in Figure 4).<sup>32</sup> After these initial sudden and unexpected shocks, we model the rest of the Great Depression through elevated unemployment risk and depressed TFP that persist through 1945. Unlike the initial shocks, these persistent aggregate shocks after 1932 are no longer treated as a surprise. We model the initial unexpected economic downturn associated with the Great Depression as one that affects the economy through three distinct channels: an adverse TFP shock, an adverse unemployment shock, and an adverse capital depreciation shock. We calibrate these shocks to match the total changes in the available empirical estimates of the TFP, unemployment rate, and capital stock between 1929 and 1932 (see time line in Figure 4).<sup>33</sup> After these initial sudden and unexpected shocks, we model the rest of the Great Depression through elevated unemployment risk and depressed TFP that persist through 1945. Unlike the initial shocks, these persistent aggregate shocks after 1932 are no longer treated as a surprise.

To estimate the TFP shock we turn to historical estimates of TFP from John (1961). Generally, with the exception of the Great Depression, Kendrick's TFP series increases throughout the first half of the 20th century. In order to isolate the change in TFP (or the TFP shock) due to the Great Depression, we control for the observed time trend by regressing Kendrick's TFP series on a third order polynomial in time and a binary indicator for the Great Depression (1930-1940). For 1932-1940, we estimate the shock to TFP as the difference between the predicted TFP (excluding the effects of the Great Depression) from the regression and the actual level of TFP from Kendrick. After 1940, one complicating factor of our analysis is the presence of the economic effects associated with WWII that were probably not anticipated at the time when Social Security was adopted.<sup>34</sup> To exclude the potential extra boost to TFP from WWII, we assume that instead of recovering immediately in 1940, TFP linearly recovers over the next five years. After 1945, the TFP is kept fixed at the 1945 value. The left panel of Figure 5 plots TFP shock that we incorporate in the model.

Next, the unemployment shock is modeled through changes in the probability of becoming unemployed,  $p^U$ . The right panel of Figure 5 plots the unemployment rates from their initial steady state level throughout the Great Depression that we derive using the available data.<sup>35</sup> Similar to TFP, we do not want to incorporate the decrease in the unemployment rates that are due to WWII, so we assume the shocks linearly dissipate starting in 1941 such that the unemployment rate returns to 4.1% by 1945.

Finally, turning to the capital depreciation shock, according to the BEA, the value of fixed assets fell by 24 percent between 1929 and 1932. We implement this shock with a one-time increase of 24 percentage points to the depreciation rate,  $\delta$ . This one-time increase in  $\delta$  is assumed to be unexpected and immediately dissipates, though its effects on the

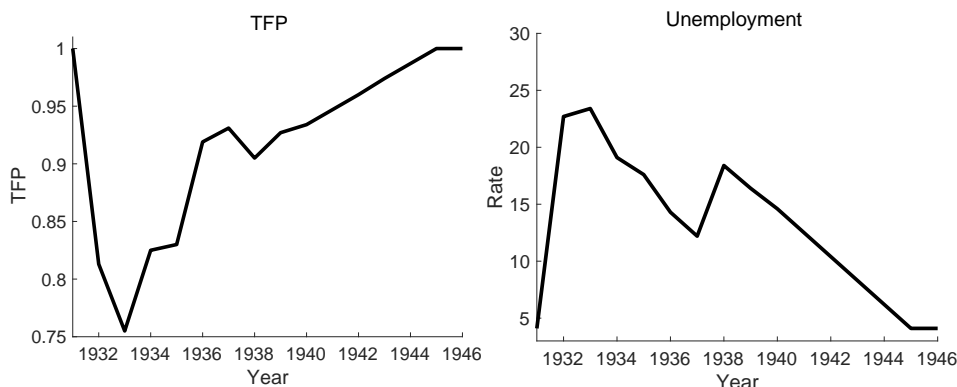
<sup>32</sup>For computational convenience, the initial 1929-1932 changes in TFP, capital stock, and unemployment are condensed into a single period.

<sup>33</sup>For computational convenience, the initial 1929-1932 changes in TFP, capital stock, and unemployment are condensed into a single period.

<sup>34</sup>Although the United States did not enter the war until later, production for war activities aboard increased prior to the U.S. entering the war.

<sup>35</sup>We use the estimated change in unemployment rates from the NBER–Conference Board, Lebergott (1964) and Darby (1976). Despite some differences caused in part by varying definitions of the unemployed, all three series indicate a similar sharp increase in unemployment 1929 and 1932 and suggest that the unemployment rate stayed elevated for the next ten years. See Margo (1993) for a description of the differences between some of these estimates.

FIGURE 5.— TFP and Unemployment During Great Depression



Note: The figure plots TFP and unemployment in the model during the Great Depression.

economy persist as it takes time for the economy to rebuild the lost capital.

## 5.2. Social Security

When considering the *original* Social Security program we focus on the program that was established in 1937 and started providing benefits in 1940.<sup>36</sup> In particular, Social Security was initially signed into law amidst the Great Depression in late 1935. According to the original law, all eligible agents were scheduled to start funding the system in 1937, with the first benefits payments being paid out in 1942. However, the 1939 amendments introduced three notable changes: (i) the program became more inclusive, (ii) eligible agents were allowed to receive benefit payments already in January 1940 (i.e., two years ahead of the initial schedule), and (iii) income earned by agents after reaching the NRA was included in the calculation of the Social Security benefits ( $b^{ss}$ ). For computational tractability, we assume that agents learn about both the original law and these later amendments at the end of 1935.<sup>37</sup> Second, we ignore further amendments after 1940 which were not part of the initially implemented program.

During the initial phase-in, the program differed from the steady state version in three important ways. First, unlike in the steady state where all agents are eligible to collect Social Security after retirement because they paid into the system, not all agents from the original cohorts were eligible for Social Security benefits. In particular, along the transition, agents who did not contribute payroll taxes through at least 1940 were ineligible for Social Security.<sup>38</sup> Second, payroll tax rates during the phase-in were lower compared to the steady state. In accordance with the historical experience, we thus set the 1940-1945 payroll tax rates equal to their historical levels (see Table III). After 1945 we let  $\tau^{ss} = 0.045$ , the rate at which the Social Security program's budget is balanced in the final steady state. Third,

<sup>36</sup>This program excludes a number of features in the current Social Security program such as indexing of benefits to ensure they reflect the general rise in the standard of living over an individual's working lifetime. If these features were included then any source of growth in wages (e.g. technological growth) would affect the return of Social Security. However, given that the indexing was not included in the original program it is excluded from our analysis.

<sup>37</sup>Therefore, prior to 1936 agents are unaware that the program will be enacted and act as if the program will not exist.

<sup>38</sup>On exception to this general rule were agents who turned 65 between 1937 and 1940. These agents paid Social Security taxes until they turned 65, but did not qualify for the standard retirement benefit calculation as described in Section 2.4. Instead, these agents were reimbursed 175% of the amount they contributed in payroll taxes in a lump sum payout. We incorporate this exception for agents who retire between 1937 and 1940 into our model.

TABLE III  
HISTORICAL PAYROLL TAX RATES

Year	Payroll Tax Rate
1937	2.0%
1938	2.0%
1939	2.0%
1940	2.0%
1941	2.0%
1942	2.0%
1943	4.0%
1944	4.0%
1945	4.0%

Note: The payroll tax rates from 1937 through 1945 are equal to their historical values. After 1945 they are set at 4.5%, consistent with the rate that clears the Social Security budget constraint in the steady state.

benefits were calculated from the average lifetime earnings the period after the program was adopted. Thus, along the transition equations 2.1 and 2.3 are altered to:

$$x_{j+1} = \frac{\min\{y_j, \bar{y}\} + (j-1-s)x_j}{j-s},$$

and

$$b^{ss} = b_{base}^{ss} \times \left(1 + \frac{R-s}{100}\right),$$

where  $s$  is the agent's age in 1937 when agents began paying payroll taxes.

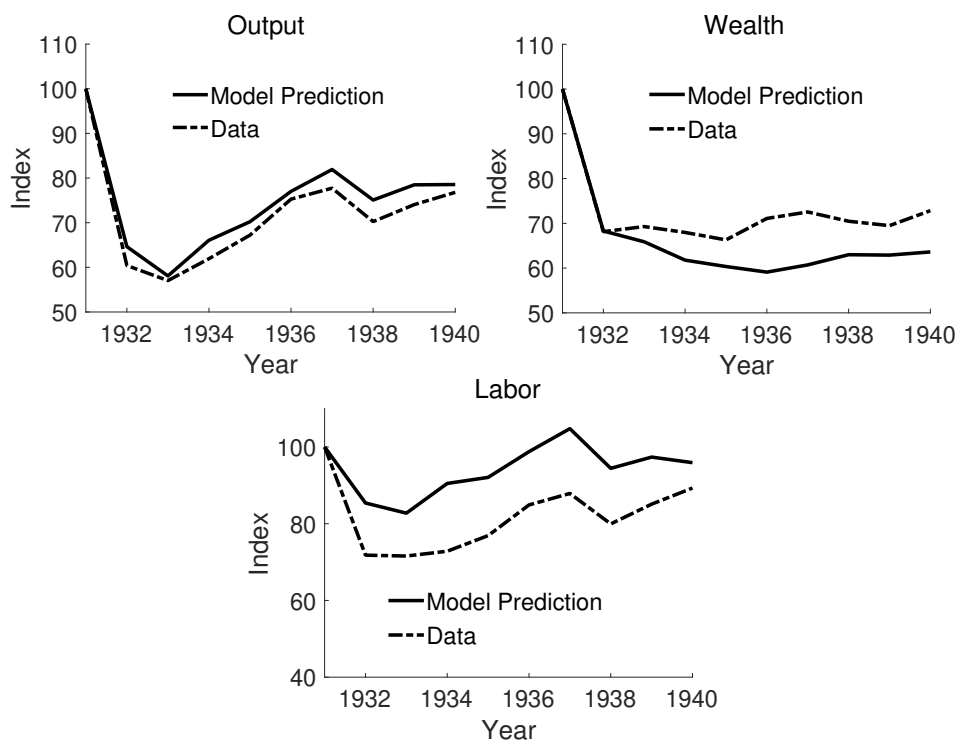
### 5.3. A Comparison of the Baseline Transitional Path to the U.S. Data

Figure 6 compares the evolution of aggregate output, wealth, and hours in baseline transition (which includes the historical events of the Great Depression and the subsequent adoption of Social Security) to the fluctuations in the actual data.<sup>39</sup> We only compare the model to the data for the first ten years after the Great Depression because in 1940 the war build-up—not captured by our model—might have begun to affect these aggregates. Moreover, during the subsequent transitional years, there were a number of changes to Social Security and fiscal policy that are not included in our experiment (see Section 4.5). Overall, the model does a good job predicting the actual fluctuations in output and wealth during the comparable period. However, the model under-predicts the fluctuations in aggregate labor, likely because the model does not incorporate under-employment during the Great Depression.<sup>40</sup>

<sup>39</sup> Appendix C examines how the evolution of these macroeconomic aggregates differs within our model when Social Security is not adopted.

<sup>40</sup> As such, our welfare results may under-predict the total harm from the Great Depression.

FIGURE 6.— Predicted Fluctuations versus Actual Fluctuations



Note: The solid lines capture the simulated changes in economic aggregates along the transition path relative to their original values in the steady state without Social Security. The dashed lines capture the actual changes in the aggregate economic variables relative to their trend. The trends are calculated using a second order polynomial using data from 1900 through 1929. All values are indexed to 100 in 1929, which is considered the steady state. The historical data series comes from John (1961).

## 6. RESULTS

This section presents our welfare results. In Section 6.1, we define our computational experiment and transitional welfare measures. Section 6.2 present our key transitional welfare results. In Section 6.3, we examine how these welfare effects differ by age and wealth. Section 6.4 conducts sensitivity analyses.

### 6.1. Transitional Welfare Measure

In order to assess the welfare effects of adopting Social Security for the original cohorts, we calculate two separate transitional paths. First, we simulate the baseline transition from the initial steady state without Social Security to the final steady state with Social Security along which the Great Depression happens. Second, we simulate a counterfactual transition in which Social Security is not adopted, but the Great Depression still occurs. Comparing the welfare of agents between these two transitional paths pins down the welfare effects from adopting Social Security.

We use two welfare metrics to gauge the transitional welfare effects from adopting Social Security for the original cohorts. First, we calculate the ex-post likelihood that an agent from cohort  $s$  will experience greater total lifetime utility in the benchmark transition in which Social Security is adopted than in the counter-factual transition in which

TABLE IV  
DECOMPOSITION OF TRANSITIONAL WELFARE EFFECTS FROM SOCIAL SECURITY

	Total Effect	Contribution From:		W/O Great Depression
		Direct Effect	G.E. Effect	
CEV <sup>T</sup>	5.7%	7.5%	-1.8%	8.2%

Note: The total welfare effects are the CEV necessary to make a cohort indifferent between living through the benchmark and counter-factual transitions. These effects are the average welfare gain across all living cohorts. The total effect is decomposed into the direct effect and G.E. effects. The W/O Great Depression measures the total welfare effect in the absence of the Great Depression.

Social Security is not adopted. We refer to this likelihood as  $\Theta$ , and define it as:

$$\begin{aligned} \Theta = \text{Probability} & \left[ u(c_s^B) + v(h_s^B, D_s) + \sum_{j=1}^{J-s} \beta^j \Pi_{q=s}^{s+j}(\Psi_q) [u(c_{s+j}^B) + v(h_{s+j}^B, D_{s+j})] \mu(\Xi) \right] \\ & > u(c_s^C) + v(h_s^C, D_s) + \sum_{j=1}^{J-s} \beta^j \Pi_{q=s}^{s+j}(\Psi_q) [u(c_{s+j}^C) + v(h_{s+j}^C, D_{s+j})] \mu(\Xi), \end{aligned}$$

with  $c_s^B$  and  $c_s^C$  denoting the per-period consumption levels in the benchmark transition and the counter-factual transition, respectively,  $s$  denoting the agent's age in 1937, and  $\mu(\Xi)$  denoting the distribution of agents across state variables.

Second, we define *transitional* CEV (or CEV<sup>T</sup>) as the uniform percent increase in *expected* consumption in each period over the *remainder* of an agent's lifetime that makes the agents from cohort  $s$  indifferent between experiencing the benchmark and the counter-factual transitions:

$$\begin{aligned} E & \left[ u(c_s^B) + v(h_s^B, D_s) + \sum_{j=1}^{J-s} \beta^j [u(c_{s+j}^B) + v(h_{s+j}^B, D_{s+j})] \right] \\ & = E \left[ u\left( \left(1 + \frac{\text{CEV}^T}{100}\right) c_s^C \right) + v(h_s^C, D_s) + \sum_{j=1}^{J-s} \beta^j \left[ u\left( \left(1 + \frac{\text{CEV}^T}{100}\right) c_{s+j}^C \right) + v(h_{s+j}^C, D_{s+j}) \right] \right]. \end{aligned}$$

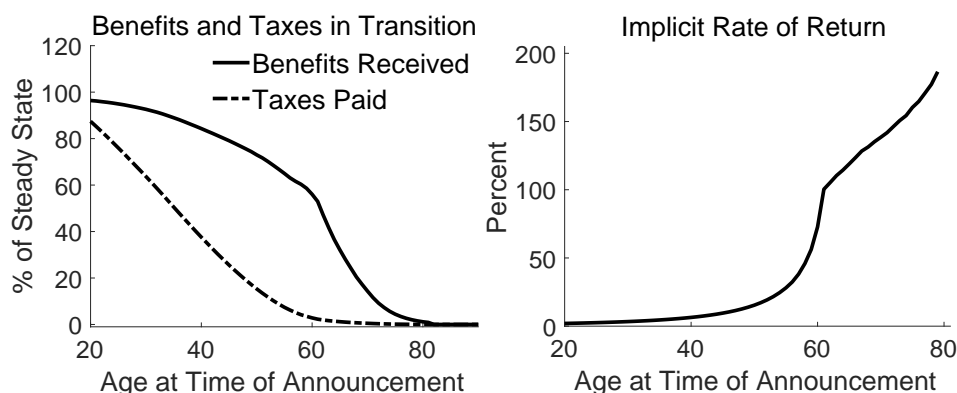
A positive CEV<sup>T</sup> implies a welfare gain from the program's adoption.

## 6.2. Transitional Welfare Effects

The first column in Table IV reports the aggregate welfare results. We start by computing CEV<sup>T</sup> associated with the adoption of the program, which is the amount of expected consumption that makes transitional agents indifferent, on average, between experiencing the baseline transition with the Great Depression and the adoption of Social Security (i.e.,  $B$  = Great Depression and Social Security) and a counter-factual transition where the Great Depression occurs but Social Security is not adopted (i.e.,  $C$  = Great Depression and no Social Security). We find a sizable average welfare gain across the cohorts from the adoption of Social Security, equivalent to a 5.7 percent increase in these agents' expected future consumption. Moreover, the ex-post likelihood that these living agents gained welfare due to the adoption of Social Security,  $\Theta$ , is 89.9 percent.

The welfare gain can be decomposed into two standard channels: the direct effect and the general equilibrium effect. The direct effects of Social Security are comprised of the welfare improvement from the inter- and intra-generational

FIGURE 7.— Effect of Gradual Implementation



Note: The values in the left panel indicate the average percent each agent pays into and receives from Social Security compared to the respective values if these agents lived in the steady state with Social Security. The values in the right panel are the average rate of return required to equate the net present value of the stream of contributions to and benefit payments from Social Security. The values in both panels are the average within a cohort.

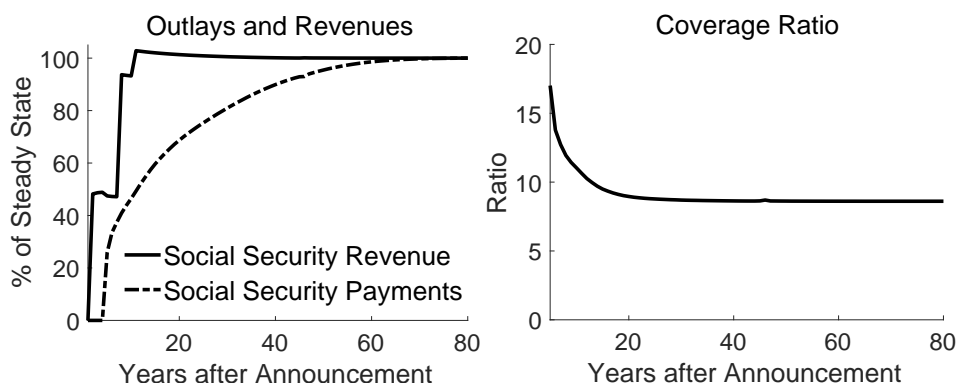
insurance. Additionally, the direct effect includes the adverse consequences of the payroll taxes, which make it harder for younger and low-wage agents to earn enough after-tax income to accumulate precautionary savings and smooth consumption, as well as the distortions to agents' labor supply decisions from the progressive contribution-benefits formula. With regard to the general equilibrium channels, the program crowds out private savings, thereby reducing aggregate capital and distorting marginal products of capital and labor in the economy. Most studies find that in the steady state Social Security reduces welfare because the negative general equilibrium effect outweighs the positive direct effect.<sup>41</sup>

In order to decompose the overall welfare effects into these channels for the transitional agents, we run a different counter-factual simulation where agents experience the introduction of Social Security and the Great Depression, but where the factor prices are set exogenously (i.e.,  $C^{FP}$  = Great Depression, Social Security, and exogenous factor prices). Specifically, to eliminate the effect of the adoption of Social Security on factor prices, in this experiment we exogenously set factor prices equal to their equilibrium values generated along a transitional path that experiences the depression but *not* the introduction of Social Security. We then calculate the welfare change,  $CEV^{FP}$ , which makes an agent indifferent between living through this alternative transition ( $C^{FP}$ ) where the adoption of Social Security has no effect on factor prices and the baseline transition ( $B$ ) where it does. Calculating the  $CEV^{FP}$  between these two transitions isolates the welfare effect implied by the change in factor prices due to the general equilibrium effects of Social Security. Moreover, subtracting this general equilibrium effect from the total welfare effect (i.e., subtracting the third column from the first column in Table IV) isolates the welfare change due to the direct effects (the second column).

As expected, Table IV shows that the general equilibrium effect contributes negatively to the overall welfare effect from Social Security, while the direct effect contributes positively. However, in contrast to the standard steady state findings, the general equilibrium effect is much smaller than the direct effects, causing the overall CEV to be positive

<sup>41</sup>Although a bit smaller due the relative size of the program (2.1 percent of expected lifetime consumption), the negative steady state welfare effects associated with the original program are qualitatively the same as those of the current program (the focus of most other studies).

FIGURE 8.— Social Security Outlays and Revenues



Note: The values in the left panel are the total outlays or revenues received in a particular year. The values are normalized as a percent of the total outlays and revenues received in the steady state with Social Security. Outlay equal revenues in this steady state. The right panel is the ratio of agents paying payroll taxes to the number of agents receiving benefits.

on net. The negative general equilibrium effect is smaller because it takes many periods for transitional agents to adjust their savings levels in response to the program's adoption and, thus, the crowd-out of capital is substantially muted over the transition (see Figure 14 in Appendix C). Moreover, the direct effect is much larger for the transitional agents than it is typical in long-run studies because the program was considerably more generous over the transition. To illustrate this, the solid and dashed lines in the left panel of Figure 7 plot the average lifetime Social Security benefits received and taxes paid (expressed as a fraction of their final steady state values) by agents in the benchmark transition (*B*). The difference between the two lines demonstrates that most agents in the economy during the transition received far more benefits relative to their Social Security contributions than what they would have received if they lived their entire life in the steady state with Social Security. Moreover, the right panel demonstrates that the more generous program is associated with a large implied internal rate of return from Social Security for most cohorts.<sup>42</sup>

The transitional agents contributed relatively less into the Social Security system for two reasons. First, the payroll tax rates were initially introduced at the low level of 2 percent (less than half of the steady state level), and stayed low for a number of years. Second, the original cohorts did not start paying into the system until the program was adopted, part way through their life. In contrast, the benefits were fully implemented immediately, though the scaling factor (based on years of employment) somewhat lowered the benefits for the transitional agents because these agents did not pay as many years into the system. Overall, this implies that the Social Security benefits were, on net, more generous relative to agents' contributions for these original cohorts.

Although the program is structured such that the taxes are more gradually implemented than the benefits, we find that the program does not run a deficit. The left panel in Figure 8 plots the total outlays and revenues for Social Security in each year after the program is announced (as a percent of their respective final steady state values). We find that in all periods revenues either equal or exceed outlays, largely because the number of individuals contributing payroll

<sup>42</sup>The return to savings fluctuates from approximately negative 20 percent to positive 7 percent in the early years of the transition, before leveling off at its final steady state value of 4.3 percent (see panel H in Figure 14 in Appendix C). For most cohorts the implicit rate of return from Social Security exceeds this average rate of return from savings. For example, over the first 20 years of the transition, the average return to savings is 4.5 percent, which is less than the implicit rate of return from Social Security experienced by cohorts over the age of 36 at the time of the reform. Specifically, the implicit rate of return from Social Security is 2, 3.4, 6.4, 15.4, and 72.8 percent for cohorts of the age 20, 30, 40, 50, and 60 at the time of the reform, respectively.

taxes exceeds the number of Social Security beneficiaries in a given period by roughly a factor of 10 (right panel in Figure 8).<sup>43</sup>

It is also interesting to explore the role of the Great Depression in our finding. On the one hand, the Great Depression could have bolstered the welfare gains because the old-age insurance from Social Security could have been more valuable during the Great Depression when large amounts of wealth and income were lost. On the other hand, imposing a payroll tax on agents during the Great Depression when agents suffered from tighter budget constraints due to the adverse shock could lower the welfare gains from the program's adoption. To this end, we calculate the transitional  $CEV^{NoGD}$  that, in the absence of the Great Depression, makes transitional agents indifferent between experiencing a transitional path wherein the Social Security program is implemented ( $B^{NoGD}$  = no Great Depression but Social Security) and a path where the program is not adopted ( $C^{NoGD}$  = no Great Depression and no Social Security). We find that introducing Social Security without the Great Depression raises welfare by the equivalent of 8.2 percent of expected future consumption; notably more than the welfare gain of 5.7 percent in the economy where the Great Depression occurs. One reason why the Great Depression exacerbates the adverse effects from introducing the payroll tax more than it increases the benefit of introducing the insurance is that most of transitional agents did not receive Social Security payments for many years to come after the Great Depression, but had to start funding the system immediately, at a time when economic conditions were especially weak.

### 6.3. Transitional Welfare Effects by Age and Wealth

In this section, we examine the welfare effects by age for agents who were eligible for the program at the time of the program's announcement; this group comprises 90 percent of the population.<sup>44</sup> Figure 9 summarizes the findings for these agents. In the left panel, the likelihood of welfare gains from the adoption of Social Security generally rises with the cohort's age at the time of the program announcement until age 50 when it reaches approximately 100 percent. Moreover, in the right panel, each cohort's expected ex-ante gain from the adoption of Social Security mostly grows exponentially with age.<sup>45</sup>

There are two main reasons why the program was more beneficial for older agents. First, individuals who were younger at the time of the program's announcement were more likely to be liquidity constrained, amplifying the adverse effects from the newly implemented payroll taxes.<sup>46</sup> Second, the older an agent was at the time of the program's adoption the fewer years of payroll taxes the agent contributed prior to receiving Social Security benefits. While fewer

<sup>43</sup>Similarly, through 1960 annual total expenditures from the Old Age Survivorship Disability Insurance (OASDI) trust fund were less than annual revenues. However, making this comparison in the data and the model is not completely equivalent for two reasons. First, both revenues and expenditures in the data include parts of OASDI other than just the old-age consumption insurance. Second, further amendments of Social Security made the program larger.

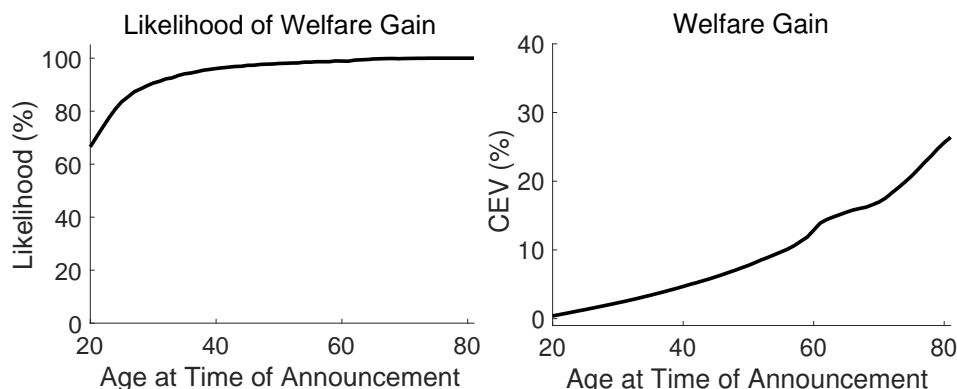
<sup>44</sup>We find that the welfare effects for ineligible agents are associated with small average welfare gains for older ineligible agents and small average losses for younger agents. Moreover, agents who enter the economy immediately after the implementation of the program experience, on average, a small welfare gain. However, as time passes the program becomes less beneficial for new entrant, eventually leading to a decrease in welfare.

<sup>45</sup>Our transitional welfare measure,  $CEV^T$ , quantifies the welfare gain from the adoption of Social Security relative to the rest-of-life welfare. However, one could measure the transitional welfare in terms of total lifetime welfare. Using total lifetime welfare, the transitional CEV is hump-shaped, reaching its peak of roughly 2.5 percent for agents who are 50 years of age at the time of the program's announcement. For further discussion about the distinction between these two measures see Auerbach and Kotlikoff (1987) (pp. 154 of their study).

<sup>46</sup>For example, in the initial steady state without Social Security, over 20 percent of all agents are liquidity constrained (i.e. have a level of savings that is smaller than 1 percent of the average level of savings in the economy). Essentially all of these liquidity-constrained agents are under the age of 35.



FIGURE 9.— Welfare Effect for Eligible Agents from Implementing Social Security by Age



Note: The values are the average within each cohort for agents that are eligible to receive Social Security benefits.

years of contributed payroll taxes lowered the post-retirement benefit size, this reduction in benefits was relatively smaller than the decrease in total payroll tax liability (see Figure 7).

Interestingly, the speed of the increase in the welfare gains by age slows temporarily for cohorts age 62 to 70. This slowdown is primarily a product of variation in welfare effects by wealth for each cohort. The upper-left panel of Figure 10 plots the welfare gain by age for each quintile of the wealth distribution.<sup>47</sup> After age 62, the welfare gains from the adoption of Social Security decline for agents in the top two quintiles. In contrast, the welfare gains continue to rise or hold steady for cohorts ages 62+ in the lowest three quintile.

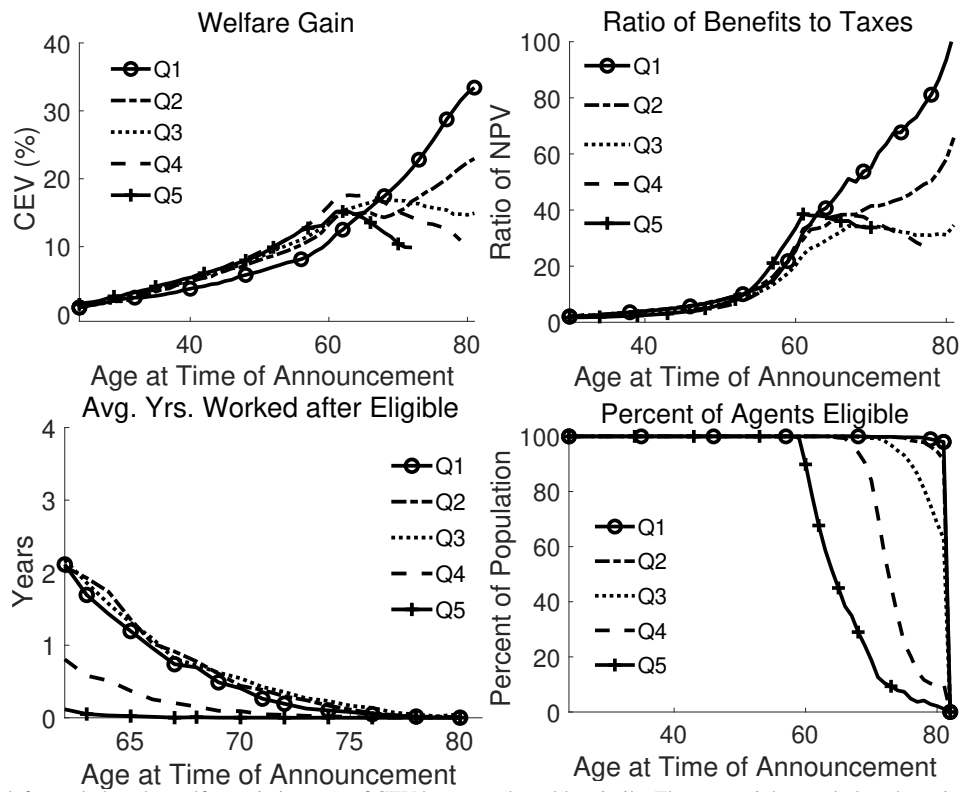
The differences in the welfare effects by wealth for each of these cohorts are driven by the variation in the discounted net present value (NPV) of the ratio of the expected benefits to payroll taxes. In particular, the upper-right panel in Figure 10 plots the ratio of the expected present value of benefits to the expected present value of the payroll taxes. Thus, a ratio above 1 indicates that the expected net present value of benefits is larger than that of taxes.<sup>48</sup> The variation in this ratio primarily arises from differences in retirement decisions across quintiles. The lower-left panel in Figure 10 plots the average number of years that a transitional agent works after becoming eligible to collect Social Security benefits by wealth quintile. For low-wealth agents, the number of years that a transitional agent works after becoming eligible to collect benefits declines with the agent's age at the time of the program's adoption. Thus, for these older low-wealth agents, the NPV of the benefits-contribution ratio tends to rise because the ratio of expected years receiving Social Security benefits versus years contributing payroll taxes rises with their age at the time of the program's announcement.<sup>49</sup> In contrast, irrespective of their age at the time of the program's announcement, agents in the top wealth quintile generally retire immediately after becoming eligible for benefits (i.e., after contributing three years of payroll taxes). As a result, the NPV of these agents' Social Security contributions is quite similar across

<sup>47</sup> Given that retirement decisions by income, we divide agents into quintiles regardless of their eligibility for Social Security in order to include a consistent composition of agents within each of the wealth quintiles across the different cohorts. Once we have segmented the population into the different quintiles, the upper-left, upper-right, and lower-left panels plot the effects for only the eligible agents within each of these quintiles. As a result, the relative size of each quintile that is eligible for the program changes with the cohort's age at the time of the program's adoption.

<sup>48</sup> The upper-right panel in Figure 10 plots  $\frac{E[\text{NPV benefits}]}{E[\text{NPV taxes}]}$ , where the NPV of expected benefits for cohort  $s$  is defined as  $E[b_s^{ss} + \sum_{j=1}^{J-s} \beta^j \Pi_{q=s}^{s+j}(\Psi_q) b_{s+j}^{ss}]$  and the NPV of payroll taxes is  $E[\tau^{ss} \min\{y_s, \bar{y}\} + \sum_{j=1}^{J-s} \beta^j \Pi_{q=s}^{s+j}(\Psi_q) \tau^{ss} \min\{y_{s+j}, \bar{y}\}]$ .

<sup>49</sup> Moreover, there is a minimum Social Security benefit but no minimum on the payroll tax, which further increases the NPV for the lowest income agents (who are concentrated in older cohorts in the lowest quintile).

FIGURE 10.— Effect by Age and Wealth



Note: The upper-left panel plots the welfare gain in terms of CEV by age and wealth quintile. The upper-right panel plots the ratio of the net present value of the lifetime benefits received from the program relative to the lifetime payroll taxes paid. The lower-left panel describes the number of years agents work after becoming eligible to start receiving Social Security benefits. The lower-right panel describes the percent of agents who are eventually eligible to receive benefits. The wealth quintiles are determined for each agent by comparing the total wealth at the time of the announcement of Social Security within each cohort for all agents regardless of their eligibility. However, we only report the welfare effects, the ratio of benefits to taxes, and average years worked for the agents that are eligible.

cohorts. Meanwhile, the NPV of the total benefits received declines the older an agent is at the time of the program's adoption primarily due to rising mortality risk.<sup>50</sup> Thus, the overall welfare gain for these high-wealth individuals' decreases the older an agent is at the time of the program's adoption.

Differences in retirement decisions also cause the share of eligible agents from each of the wealth quintiles to vary across the cohorts. Since higher-wealth individuals tend to retire earlier, cohorts who are older at the time of the announcement are disproportionately made up by low-wealth agents (see lower-right panel in Figure 10). Thus, among cohorts who are in their sixties at the program's announcement, the fraction of wealthy agents, whose  $CEV^T$  decreases with age, is large enough to cause a slowing in the increase in the aggregate  $CEV^T$ . However, among cohorts who are in their seventies, the lower wealth quintile makes up a large enough fraction of the eligible agents in these cohorts so that the  $CEV^T$  rises at an increasing speed.

Overall, Figure 10 illustrates that there is a considerable degree of heterogeneity in the welfare effects within certain cohorts. The two primary sources of within cohort heterogeneity are per-period idiosyncratic labor productivity risk and innate ability types. In order to quantify their respective contribution to our welfare findings we calculate the welfare effects in a model that eliminates the idiosyncratic labor productivity risk leaving only innate ability as the source of within-cohort heterogeneity. The left panel of Figure 11 separates these welfare effects in the economy without idiosyncratic risk for high and low ability types. Similar to the model with idiosyncratic labor productivity risk, there is little heterogeneity within younger cohorts. Although the gap in welfare effects starts to widen between low and high ability agents for cohorts near retirement age, without the idiosyncratic labor productivity risk the degree of within-cohort heterogeneity is considerably more subdued than in the model which includes idiosyncratic risk. Thus, idiosyncratic labor productivity shocks play an important role in generating the high degree of within-cohort heterogeneity in the welfare effects observed for certain cohorts in the upper left panel of Figure 10. Despite the different degree of within-cohort heterogeneity, the right panel of Figure 11 demonstrates that the contours of the average welfare gains for each cohort are similar in the economies with and without the idiosyncratic risk, generally increasing with the cohorts' age.<sup>51</sup>

#### 6.4. *Sensitivity Analysis*

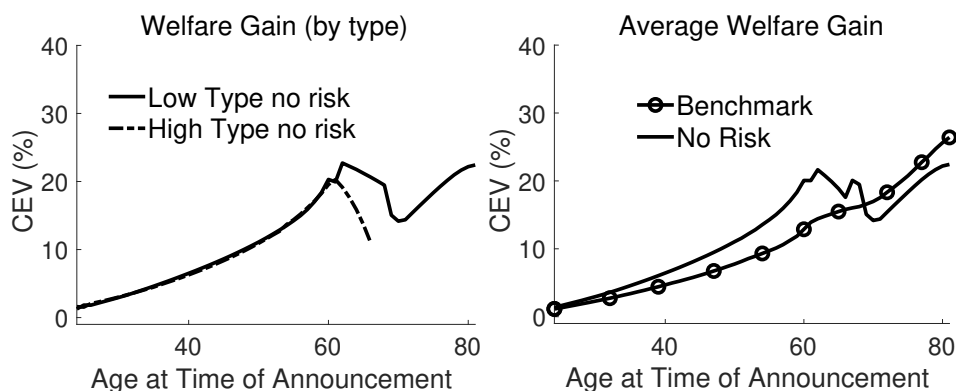
Finally, we test sensitivity of our results along six dimensions. Table V presents both the expected average level and ex-post likelihood of welfare gains for transitional agents for each experiment. As is the case in the rest of this paper, the welfare results reported in the table are derived from an experiment that compares welfare in the baseline transition (*B*) in which both the Great Depression and the adoption of Social Security occur to the welfare in the counter-factual transition (*C*) in which the Great Depression takes place but Social Security is not adopted.

In the first experiment, we compute the welfare effects in a scenario where only agents under age 65 at the time of the program's announcement are eligible to participate in Social Security. This experiment is of interest because the

<sup>50</sup>The decline in the NPV is because the older an agent is, in expectation, the fewer years he has to live and to collect the benefits. Moreover, on average older agents receive lower wages causing the eventual Social Security payment to be lower at the time of retirement.

<sup>51</sup>One small difference is that the slowdown in the welfare gains between agents 62 and 70 is relatively more pronounced in the economy without idiosyncratic risk.

FIGURE 11.— Welfare Effects With and Without Idiosyncratic Risk



Note: The values are the averages within each cohort for agents that are eligible to receive Social Security benefits. The benchmark model includes idiosyncratic labor productivity risk and the counter-factual model does not include this risk. The left panel plots the effects by cohort and ability in the model without idiosyncratic risk. The right panel plots the total effects by cohort in the models with and without idiosyncratic risk.

TABLE V

## SENSITIVITY EXERCISES

	CEV <sup>T</sup>	⊖
Benchmark	5.7%	89.4%
65+ Excluded	3.5%	76.5%
Duration Shock	6%	90.9%
Unemployment Insurance	9.9%	96.4%
Immediate Adoption	6%	84.8%
Unemployment Parameter	5.5%	89%
Frisch Parameter	4.3%	83.1%

original law announced in 1936, but amended before payroll taxes and benefits payments began, made agents over the age of 65 ineligible to participate in Social Security. In contrast, the 1939 amendments (reflected in our baseline experiments and discussed in Section 5.2) expanded the program eligibility to all non-retired agents. When the program eligibility is restricted to agents who are under the age of 65 at the time of the program's announcement, the welfare gains from the program adoption are reduced relative to the baseline results, largely because fewer older agents (for whom the welfare gains from the program's adoption are the largest) are eligible for the retirement benefits.

In the second experiment, we allow for increases in the duration of the unemployment spell during the Great Depression to reflect the actual increases observed in the available data for the Philadelphia labor market (see Table VI). Specifically, instead of holding duration constant at  $d = 0.3$ , we allow it to increase to a full year through 1941, before letting it recede linearly to its benchmark level by 1945.<sup>52</sup> The experiment results in a small net increase in welfare, due to two partially offsetting forces. On the one hand, increasing the duration of the unemployment shock tightens budget constraints, increasing the burden from payroll taxation. On the other hand, the insurance from the program becomes more valuable when the spells become longer. On net, the insurance channel dominates by a small margin. Overall, however, we find that the results are mostly insensitive to changes in the duration.

<sup>52</sup>For tractability, the duration is capped one year, as allowing the duration to last more than one period in the model would require an increase in the size of the state space vector to include whether agents were unemployed in the previous period.

In the third sensitivity experiment, the Social Security program is expanded to include a reduced-form unemployment insurance that replaces 35 percent of average earnings in the economy.<sup>53</sup> Once the unemployment insurance is included, the program contains an extra insurance component targeted towards working-aged agents, leading to even greater and more wide-spread welfare gains for the initial cohorts.

In the fourth sensitivity experiment, we compute the welfare effects under an alternative scenario wherein Social Security is adopted immediately at the onset of the Great Depression, as opposed to in the midst of it. Our welfare results are largely unchanged when Social Security is counter-factually adopted at the onset of the Great Depression, suggesting that an earlier implementation time-line for Social Security would have had only very modest effects on household welfare.

TABLE VI  
UNEMPLOYMENT DURATION DURING GREAT RECESSION (PHILADELPHIA)

1929	30%
1931	69%
1932	94%
1933	134%
1934	–
1935	211%
1936	210%
1937	213%

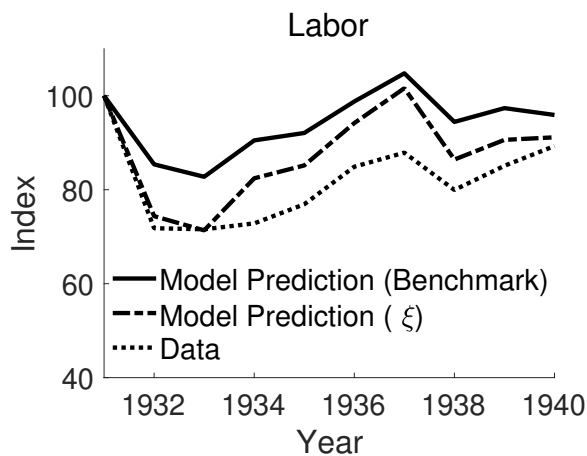
Note: Duration of unemployment is reported in percent of the year unemployed. No value for 1934 is reported.

The final two analyses examine the sensitivity of the results to alternative values for two specific preference parameters:  $\xi$  (which determines the disutility of unemployment) and  $\sigma$  (the Frisch labor elasticity). In both scenarios, we vary the parameter in which we are testing the sensitivity of the results and additionally recalibrate all the other parameters to match the specified targets. Turning first to  $\xi$ , Figure 6 shows that the fall in labor in the benchmark transition undershoots the data. Thus, in the fifth sensitivity experiment, we depart from setting  $\xi$  to the value estimated in Kaplan (2012) and instead calibrate it such that the fall in hours during the Great Depression more closely matches the data. In this alternative calibration the drop in hours worked during the Great Depression in the model aligns well with the data (Figure 12), without having much discernible effect on our main welfare findings.<sup>54</sup>

Finally, given that the estimates of the Frisch elasticity used to calibrate the preference parameter  $\sigma$  are derived from data subsequent to our study, in the sixth experiment we test the sensitivity of our results to this parameter. In particular, we recalibrate the model using a value that implies the Frisch elasticity is one-half as large as in the benchmark model (i.e.,  $\sigma = 0.25$ ). The welfare benefit of introducing Social Security is somewhat smaller with this alternative calibration because hours are less responsive to the fall in wages during the Great Depression. In particular, the insurance benefit from Social Security is somewhat less valuable because hours and labor income both fall less during the Great Depression compared to the benchmark transition.

<sup>53</sup>Between 1943 and 1960, the ratio of the average unemployment benefit within the economy compared to the average earnings was 35%. See The Employment and Training Financial Data Handbook 394 Report from the United States Department of Labor.

<sup>54</sup>Although altering  $\xi$  allows the model to more closely match the drop in hours at the onset of the Great Depression the model still over predicts the speed at which hours recover.

FIGURE 12.— Predicted vs. Actual Hour Fluctuations—Sensitivity to the Unemployment Parameter  $\xi$ 

## 7. CONCLUSION

This paper quantifies the welfare effects of Social Security for transitional agents who experienced the program's adoption. We find that the adoption of the program benefited a vast majority of these transitional agents. In particular, we estimate that the program benefited households alive at the time of the program's adoption with a likelihood of almost 90 percent, and increased these agents' welfare by the equivalent of 5.7 percent of their expected future lifetime consumption. The result that the program was quite beneficial for living agents is robust to a number of sensitivity analyses.

Through a quantitative decomposition of the overall welfare effects, we find that the adoption of the program was beneficial because most transitional agents received far greater monetary benefits in a form of Social Security payments than the amount they contributed to the system through payroll taxes. Moreover, the standard negative general equilibrium welfare effect of Social Security associated with capital crowd-out was also smaller during the transition than in the steady state, largely because it took many periods for agents to adjust their savings levels in response to the program's adoption. Moreover, we find that adopting the program in the midst of the Great Depression had only a modest effect on the welfare implications of the program's adoption and, if anything, reduced the welfare gains from Social Security for the transitional agents.

This paper highlights that the welfare implications for agents alive when the program is adopted were quite different than the steady state welfare effects. Overall, the divergent welfare benefits for agents who experienced the program's enactment versus those experienced by agents born into the steady state with Social Security might offer one explanation for why a program that potentially reduces welfare in the steady state was originally adopted.

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## APPENDIX A: EQUILIBRIUM

Here we define a stationary steady state competitive equilibrium with Social Security. An agent's state variables,  $\Xi$  are assets ( $a$ ), average past earnings ( $x$ ), age ( $j$ ), ability ( $\alpha$ ), persistent shock ( $v$ ), unemployment shock ( $D$ ), retirement status ( $I$ ). For a given set of exogenous demographic parameters ( $n, \Psi_j$ ), a sequence of age-specific productivity ( $\{\theta_j\}_{j=1}^{\bar{R}}$ ), government tax function ( $T : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ ), Social Security tax rate  $\tau^{ss}$ , Social Security benefits formula ( $B^{ss} : \mathbb{R}_+ \times j \rightarrow \mathbb{R}_+$ ), a production plan for the firm ( $N, K$ ), and a utility function ( $U : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ ), a steady state competitive equilibrium consists of agent's decision rules for  $c, h, a$ , and  $I$  for each state variable, factor prices ( $w, r$ ), transfers ( $Tr$ ), and the distribution of individuals  $\mu(\Xi)$  such that the following holds:

1. Given prices, policies, transfers, and initial conditions the agent solves the dynamic programming problem in equations 3.1 - 3.6, with  $c, h, a'$ , and  $I$  as the associated policy functions.
2. The prices  $w_t$  and  $r_t$  satisfy

$$r_t = \zeta A \left( \frac{N_t}{K_t} \right)^{1-\zeta} - \delta$$

$$w_t = (1 - \zeta) A \left( \frac{N_t}{K_t} \right)^{\zeta}.$$

3. The Social Security policies satisfy:

$$\sum \min\{w(1-D)\omega h, \bar{y}\} \tau^{ss} \mu(\Xi) = \sum b^{ss} I \mu(\Xi).$$

4. Transfers are given by:

$$Tr = \sum (1 - \Psi_j) a \mu(\Xi).$$

5. Government budget is balanced:

$$G = \sum T^y [r(a + Tr) + w(1-D)\omega h - .5\tau^{ss} \min\{w(1-D)\omega h, \bar{y}\}] \mu(\Xi).$$

6. Markets clear:

$$K = \sum a \mu(\Xi), N = \sum (1-D)\omega h \mu(\Xi) \text{ and}$$

$$\sum c \mu(\Xi) + \sum a \mu(\Xi) + G = AK^{\zeta} N^{1-\zeta} + (1 - \delta)K.$$

7. The distribution of  $\mu(\Xi)$  is stationary, that is, the law of motion for the distribution of individuals over the state space satisfies  $\mu(\Xi)(1+n) = Q_{\mu}\mu(\Xi)$ , where  $Q_{\mu}$  is a one-period recursive operator on the distribution.

The balanced growth path is a competitive equilibrium in which all aggregate variables grow at the same rate as output. Thus, all per capita variables and functions are constant, and aggregate variables grow at a constant rate of  $n$ , where  $n$  is the constant rate of population growth.

## APPENDIX B: SOLUTION ALGORITHM AND ACCURACY

To determine the steady state equilibrium, we use a modified algorithm based on algorithm 6.2.2 in Heer and Maussner (2009) for computing a stationary equilibrium for the overlapping generations model. The algorithm consists of the following steps:

1. Make initial guesses of the steady state values of the aggregate variables (e.g. capital, labor, and accidental bequests), market clearing tax rates, and the Social Security benefits in the steady state with Social Security.
2. Solve for the factor prices using the marginal product of capital and labor.
3. Compute the value function for agents on the state space of ability, idiosyncratic shocks, savings, age, average lifetime earnings, and retirement status using backward induction.<sup>55</sup>

<sup>55</sup>We discretize the savings and average lifetime earnings grids and interpolate the value function between grid points.

4. Simulate the life cycles of 3,000 agents to calculate the distribution of agents across the state space. Each agent enters the model with zero capital and faces its own unique set of idiosyncratic shocks. We draw these individual shocks from distributions consistent with our labor productivity and unemployment processes. Given these shocks and the policy functions for labor, consumption, savings, and retirement (from the value function in step 3), we iterate forward to solve for the time paths of the choice variables for each agent over his life cycle.
5. Compute the tax rates that clears the government budget constraints. Integrate over the distribution of agents to calculate aggregate variables such as capital, labor, and accidental bequests.
6. Check if the tax rates and the aggregate variables calculated in step 5 are within the tolerance of guesses in step 1. If the difference is larger than the tolerance, then update the guesses in step 1 using a weighted average of the previous guess and the new values from step 5 and return to step 2.

Once we have calculated the initial and final steady states using the previous algorithm, we use a shooting algorithm based on Heer and Maussner's algorithm 7.1.1 to compute the transition path between these steady states:

1. Set the number of transition periods to 100.<sup>56</sup>
2. Guess a time path for the transition of the aggregate variables (e.g. capital, labor, and accidental bequests).
3. Solve for path of factor prices.
4. Compute the value function for  $t = T - 1$  using the value function in the final steady state as the continuation value for period  $t = T$ . Continue to iterate backwards in time,  $t = T - 2$ ,  $t = T - 3$ , and so on using the previously solved value function as the continuation value.
5. Use the distribution of agents in the initial steady state to initialize the distribution of agents across the state space for time period  $t = 1$ .
6. In each period  $t > 1$  of the transition, simulate the life cycles of 3,000 agents to calculate the distribution of agents across the state space in that period. Each agent enters the model with zero capital and faces its own unique set of idiosyncratic shocks. We generate the individual shocks that are consistent with our labor productivity and unemployment processes. Given these shocks and the policy functions for labor, consumption, savings, and retirement (from the value function in step 4), and the time paths for the factor prices (from step 3), we iterate forward to solve for the time paths of the choice variables for each agent over his life cycle.
7. Integrate the individual values of capital and labor over the distribution of agents in each time period of the transition to compute the time paths of the aggregates such as capital, labor, and accidental bequests.
8. Check if the aggregate variables calculated in step 7 are within the tolerance of the guesses for each period from step 2. If the difference is larger than the tolerance, update the guesses in each period using a weighted average of the previous guesses and the new values solved for in step 7. Return to step 2.

Although Kirkby (2017) provides some discussion of the existence of equilibrium transitional paths, no proof exists for our specific model.<sup>57</sup> Thus, it is useful to provide an accuracy analysis. We use the standard Euler equation accuracy test (for example see Section 3.4 of Den Haan (2010)). In particular, assume  $c(\cdot)_{i,j,t}$  is the consumption value associated with agent  $i$ , at age  $j$ , and time  $t$  from our numerical solution. Let  $\bar{c}(\cdot)_{i,j,t}$  be the consumption choice implied by the calculated conditional expectation and the inverted Euler equation. We define the Euler equation error for that agent at age  $j$  and time  $t$  as  $100 \times \left| \frac{c(\cdot)_{i,j,t} - \bar{c}(\cdot)_{i,j,t}}{c(\cdot)_{i,j,t}} \right|$ . Figure 13 examines the distribution of these errors from the transitional path which includes the introduction of Social Security.<sup>58</sup> Generally, the size of these errors are small. In particular, examining the population-weighted distribution of the errors pooling across the entire transition, we find that over 95 percent of the distribution of the errors are less than one-tenth of a percent. Moreover, we find that less than 0.5 percent of the total errors in consumption are larger than one percent.

#### APPENDIX C: TRANSITIONAL DYNAMICS OF AGGREGATES

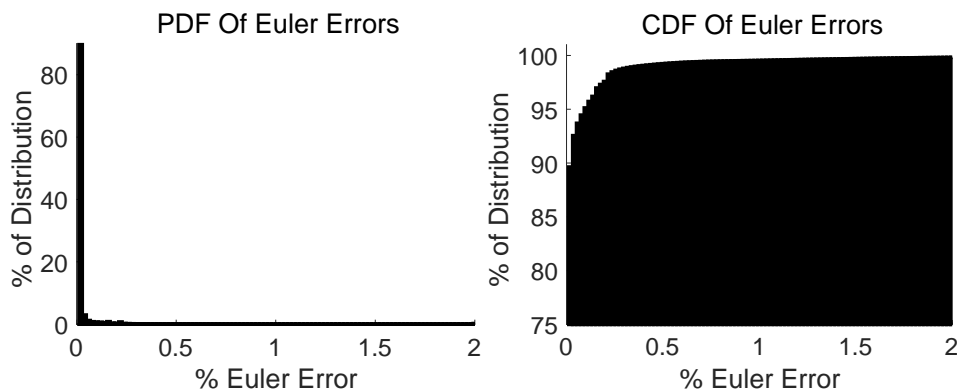
This section examines the benchmark transition of the economy from the steady state without Social Security to the new steady state with Social Security. The black lines in Figure 14 plots the transition of output, capital, labor, hours, consumption, rental rate, and wage, respectively, over the

<sup>56</sup>We check whether this is a sufficient number of periods and find that the transition occurs in less than 100 periods.

<sup>57</sup>One complicating factor is the retirement decision. In order to find an equilibrium it is necessary for us to assume that there is a continuum of agents.

<sup>58</sup>Generally, we find that the Euler equation errors without Social Security are smaller and thus only present accuracy results in the model with Social Security.

FIGURE 13.— Transitional Euler Errors



Note: The left panel plots the population weighted probability distribution function for Euler errors over the transition. The right panel plots the cumulative distribution function of these errors.

transition. Even though by 1945 the business cycle shocks dissipate and the Social Security program is fully implemented, the economy does not complete its transition to the new steady state for approximately an additional 25 years (i.e., until the year 1970).

Over the transition, aggregate output, aggregate capital, aggregate consumption, and the wage rate all fall drastically immediately upon the shock's impact, continue to decline for a few extra periods, and then gradually transition back to their new steady state values. The remaining aggregates—labor, hours, and the rental rate—suffer two sharp declines over the transition before eventually ending up at their new steady state values. The fluctuations in the aggregate economic variables over the transition come from two channels: (i) the economic shocks associated with the Great Depression, and (ii) the adoption of Social Security. In order to decompose these two effects, Figure 14 determines the percentage changes in the aggregate economic variables relative to their initial values in the steady state without Social Security under two alternative transitions. First, the dotted lines plot the evolution of the aggregates in a counter-factual transition when the economy suffers through the Great Depression but Social Security is not adopted. Second, the dashed lines describe the evolution of the aggregates in a second counter-factual transition when Social Security is adopted but there is no business cycle episode.

Turning to Panels A, B, E, and G of Figure 14, the fluctuations in the benchmark transition (black line) and the transition which only includes the Great Depression (dotted line) are similar for output, capital, consumption, and wages during the first 15 years of the transition. In these transitions, the initial declines in output, capital, consumption, and wages and the subsequent recovery are primarily caused by the shocks associated with the Great Depression. The subsequent fluctuations in these aggregates in the benchmark transition and the counter-factual transition which only includes the business cycle fluctuations tend to diverge. These later fluctuations are primarily driven by the adoption of Social Security and not the shocks to savings, TFP, and the unemployment rate.

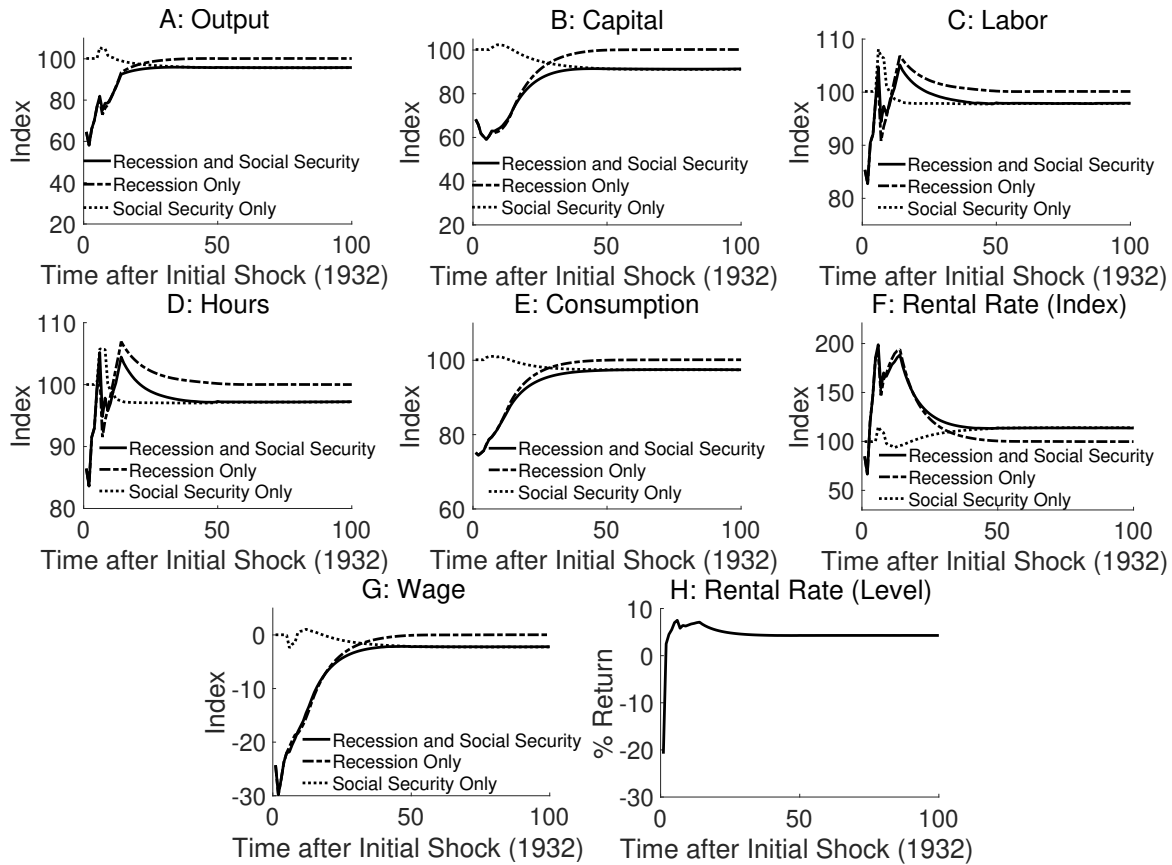
Turning to Panels C, D, and F, the transition of labor, hours, and the rental rate has multiple peaks and troughs.<sup>59</sup> Comparing the fluctuations of these three aggregates over all three transitions, the original declines are primarily driven by the business cycle shocks. The initial fall in all three aggregates is due to the drop in TFP and increase in the unemployment rate,<sup>60</sup> while the quick initial recoveries in these aggregates are due to the decline in the size of the shocks and also due to the implementation of Social Security (see the dotted and lines in Figure 14).<sup>61</sup> In particular, as the unemployment rate declines and TFP increases, agents tend to increase their hours. Additionally, in these first few periods after Social Security is announced, older agents increase their future Social Security benefit by working more. Both of these factors drive up the aggregate labor supply and rental rate. However, these increases are short-lived, as the increase in the unemployment rate in period 7 of the transition (corresponding to 1938) causes a second fall in aggregate hours, aggregate labor, and the rental rate. The second spike occurs in period fourteen. Since this spike is primarily due to the business cycle episode (the shocks to unemployment and TFP shocks finally recede), it does not occur in the counter-factual transition

<sup>59</sup>Hours capture the total number of hours worked in the economy, whereas labor is the effective labor for production (i.e. the product of hours and household productivity). Overall, the quantitative difference in the fluctuations between hours and labor are small since the distribution of hours over the idiosyncratic productivity values does not change substantially over the transition.

<sup>60</sup>The fluctuations in the rental rate are primarily driven by the changes in the ratio of aggregate labor to output.

<sup>61</sup>Note that unemployment temporarily decreases over this period but increases again in period 7 (1938).

FIGURE 14.— Aggregate Fluctuations Over Transition



Note: The solid lines capture the changes in economic aggregates along the transition path from the original steady state without Social Security to the new steady state with Social Security during the Great Depression. The dotted lines capture the changes in economic aggregates along the transition path when the economy suffers the Great Depression but Social Security is never implemented. The dashed lines capture the changes in the economic aggregates along the transition path when Social Security is adopted but there is no Great Depression. All the values in panel A-G are indices of the aggregate relative the initial value in the steady state without Social Security. Panel H plots the net return to savings not as an index but in percentage points.

without the shocks (see the dashed line in Figure 14). After the second spike in labor, hours, and the rental rate, all three aggregates slowly decrease for another 25 periods when they reach their new steady state values which are lower due to the implementation of Social Security.

#### APPENDIX D: QUALIFYING THE IMPORTANCE OF ENDOGENOUS LABOR SUPPLY

As discussed in Section 6.2, the welfare gains from the adoption of Social Security for the original cohorts stem from the fact that the NPV of Social Security benefits exceeds that of contributions for the original cohorts. Since our baseline model allows for endogenous response of labor hours and retirement decisions to the introduction of Social Security, the ratio of benefits to contributions (in NPV terms) will differ from one produced by a model without endogenous labor supply response. In this section, we explore the quantitative importance of allowing for the endogenous labor supply response, by comparing the ratio of average benefits to contributions produced by our baseline model to one that is derived holding retirement and labor decisions fixed to those observed in the initial steady state without Social Security.<sup>62</sup>

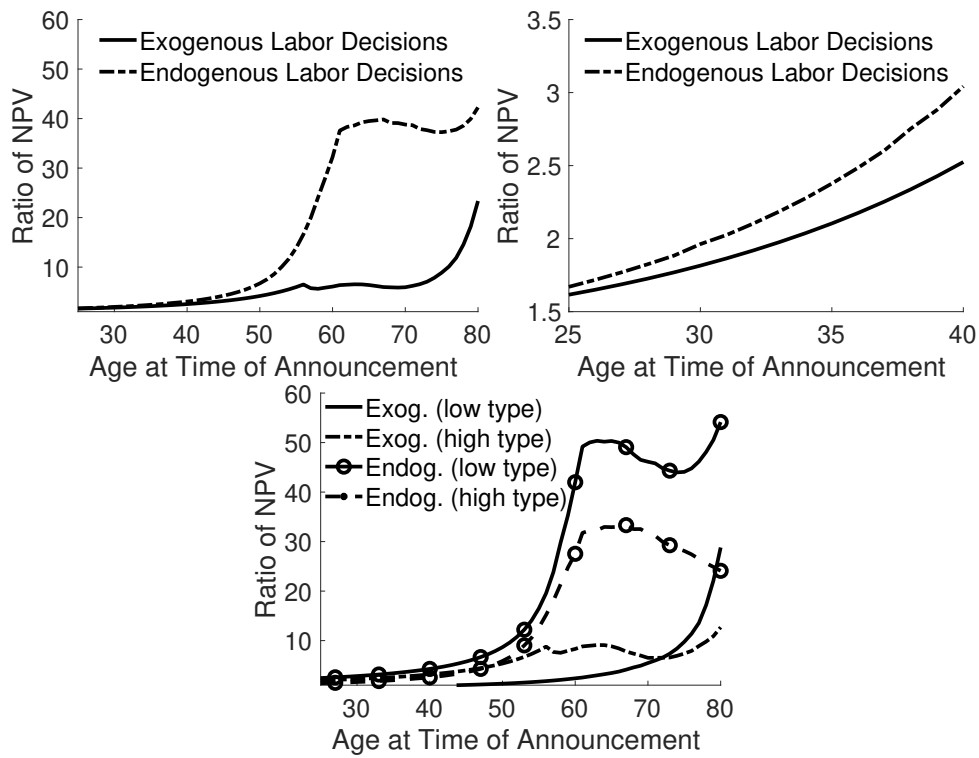
Figure 15 displays the key results. The upper-left panel shows the ratios of average benefits to average contributions (in NPV terms) produced by the endogenous vs. exogenous labor supply frameworks for all ages at the program's announcement (i.e., ages 20-80), while the upper-right panel zooms in on ages 25-40 only. The bottom panel breaks out the effects separately for high and low types. As the two panels show, the ratio of Social Security benefits to contributions produced by the baseline model with endogenous labor response is always above that generated by the alternative calculation where labor supply is not allowed to adjust in response to the enactment of Social Security. In our benchmark model, agents accelerate their retirement once Social Security is introduced, thereby contributing relatively less Social Security taxes and reaping more years of benefits than the same agents who do not adjust their labor supply in response to the program's adoption.<sup>63</sup> This effect is particularly pronounced for agents who are old enough so that they can retire (and start collecting benefits) after paying just a few years of taxes. Moreover, we find that this difference is even more pronounced for low types as opposed to high types (see bottom panel) since these are the agents that tend to make the largest adjustments to their retirement decisions in response to the introduction of Social Security. Our experiments lead us to conclude that endogenizing labor supply decisions (and retirement decisions in particular) is a quantitatively important feature of our model, particularly for older and lower ability agents.

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<sup>62</sup>To demonstrate that the discrepancy in the results is driven by the labor supply response to the introduction of Social Security rather than the Great Depression, the calculations (shown in Figure 15) are based on the transitional path where the Great Depression does not occur. However, the effects of the endogenous decisions are similar when the Great Depression is considered; these results are available at request.

<sup>63</sup>The ratio rises with age at the program's announcement largely mechanically, as the number of years of contributed taxes falls with this age, whereas the number of years during which benefits are received is unchanged, all else equal. However, the increase in the ratio of benefits to contributions is relatively larger for agents in the baseline model with endogenous labor supply who retire earlier. The larger increase arises because the same-size reduction in the number of years of contributing Social Security taxes represents a larger reduction in the fraction of total years paying taxes.

FIGURE 15.— Ratio of Benefits to Contributions



Note: The lines are the ratio of the net present value of benefits to contributions for the models with endogenous labor and when the labor decisions are fixed so they do not incorporate the addition of Social Security.